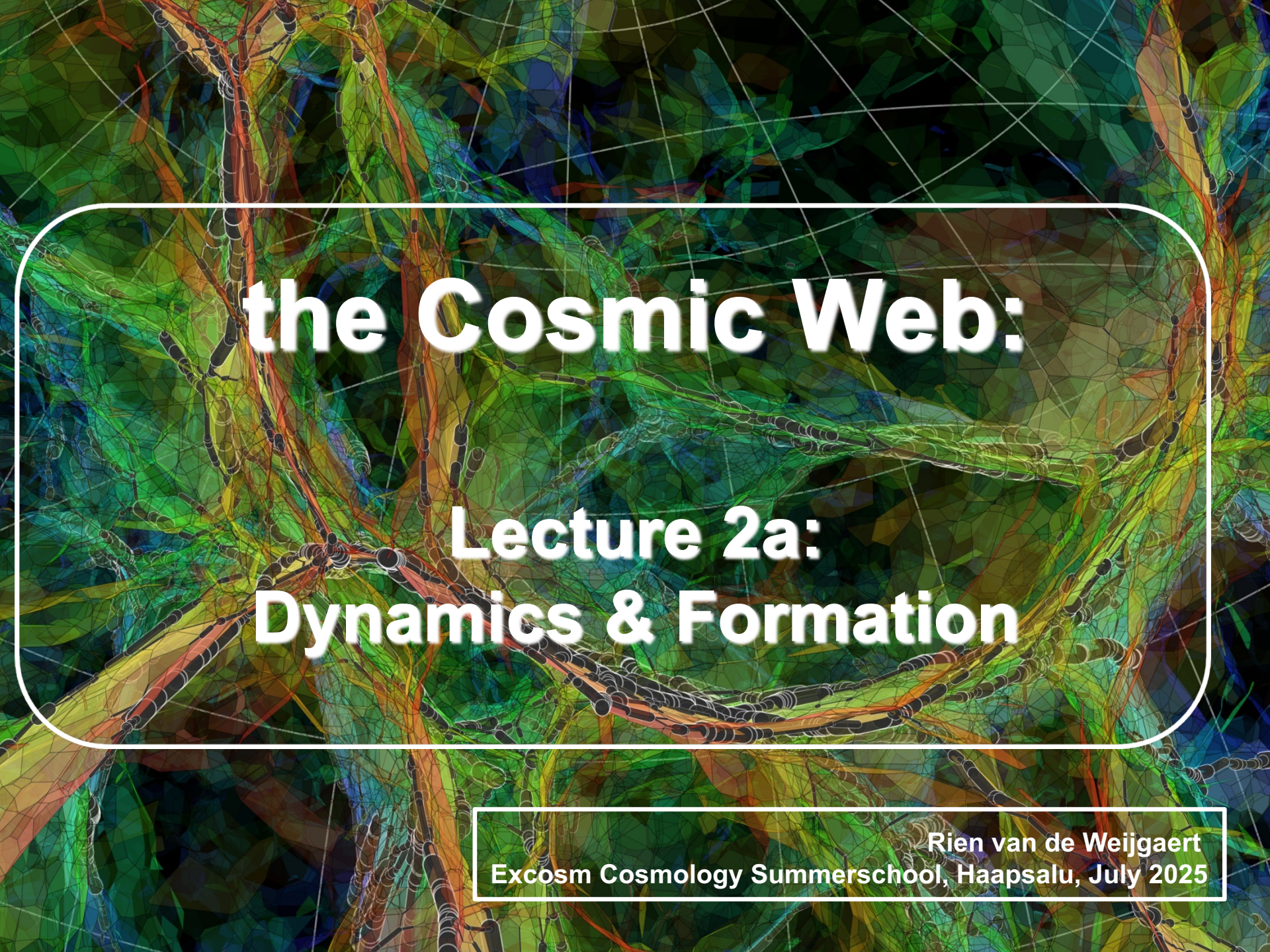


A complex visualization of the cosmic web, showing a dense network of filaments and nodes. The filaments are colored in vibrant shades of green, yellow, and orange, while the nodes are marked with black dots. The background is a dark, textured field of green and blue, suggesting the distribution of dark matter and gas in the universe.

the Cosmic Web:

Lecture 2a: Dynamics & Formation

Rien van de Weijgaert
Excsm Cosmology Summerschool, Haapsalu, July 2025

Cosmic Web

**Dynamics & Formation:
Program**

Cosmic Web – Formation & Dynamics

- **Forces & Strains** - **Observational Manifestations**
- **the Mechanism** - **Gravitational Instability**
- **Anisotropic Collapse** - **Formation of filaments and walls**
- **Weaving the Web** - **Connection Clusters, Filaments and Walls**
- **Dynamical Inventory** - **Forces & Tides in the Cosmic Web**
- **Phase Space Dynamics** - **Phase Space & Multistream structure**
- **Lagrangian Dynamics** - **Zeldovich formalism**
- **Hierarchical Formation** - **from small to the Megaparsec Cosmic Web**
- **Anisotropy & Hierarchy** - **the Adhesion formalism**
- **Caustic Skeleton** - **analytical formalism cosmic web**

Forces and Strains

in/of the Cosmic Web

Manifestations

Forces and Strains: Manifestations

- **Megaparsec Cosmic Flow field**
- **Void outflow**
- **Galaxy spin – Filament alignments**

Flow in the Cosmic Web



Cosmic Web Flowlines:

A visualization of the Cosmic Web flowlines, showing a complex network of blue and yellow streamlines with arrows indicating the direction of flow. The background is a deep blue with scattered yellow and orange spots, representing the distribution of matter in the universe.

Stokes: flow field components

Divergence
Shear

dominant in voids
dominant along filaments

Vorticity:

only in high-density
multistream regions

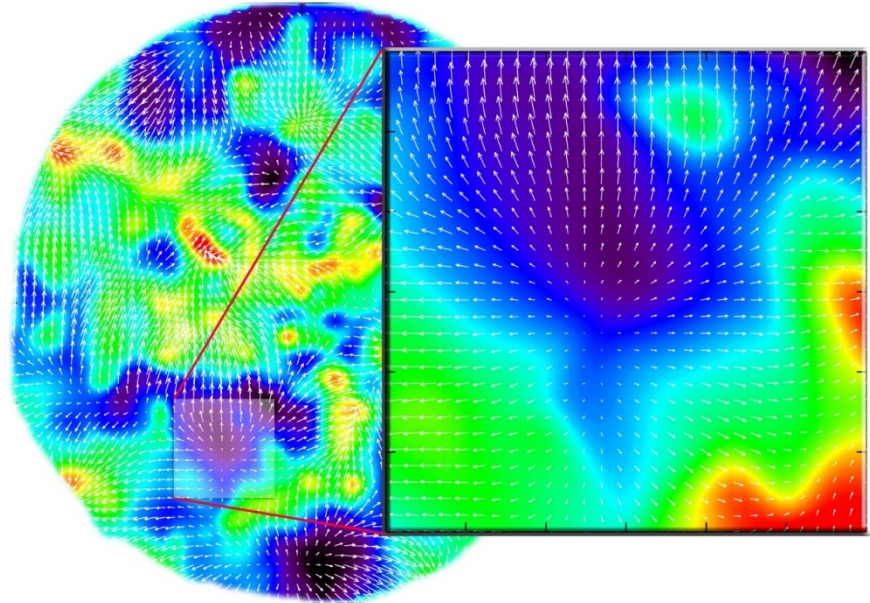
Large Scale Flow Field

Local Universe

Flow Field

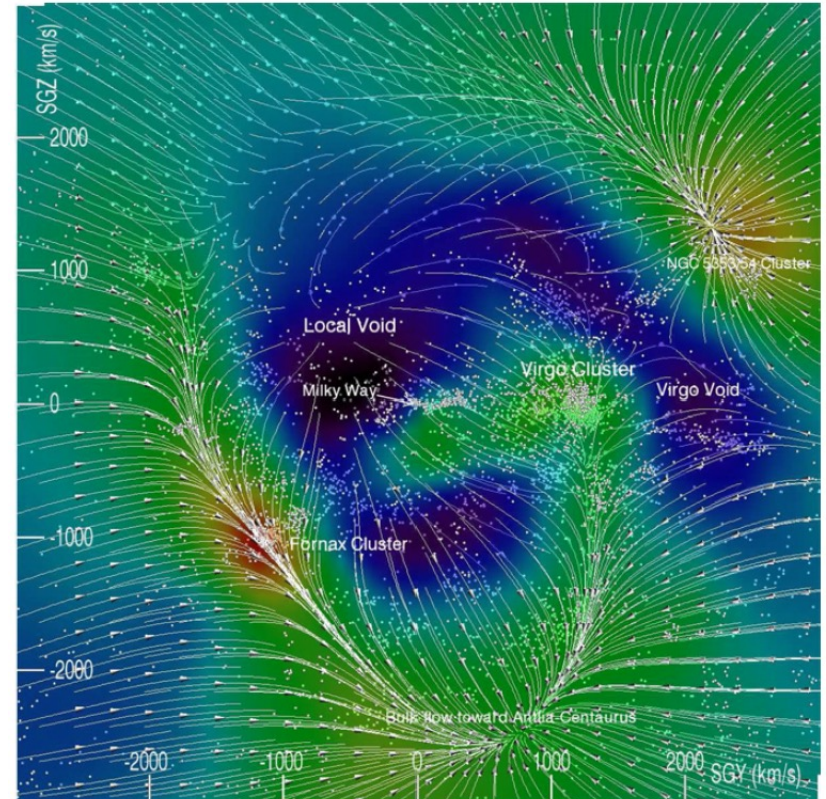
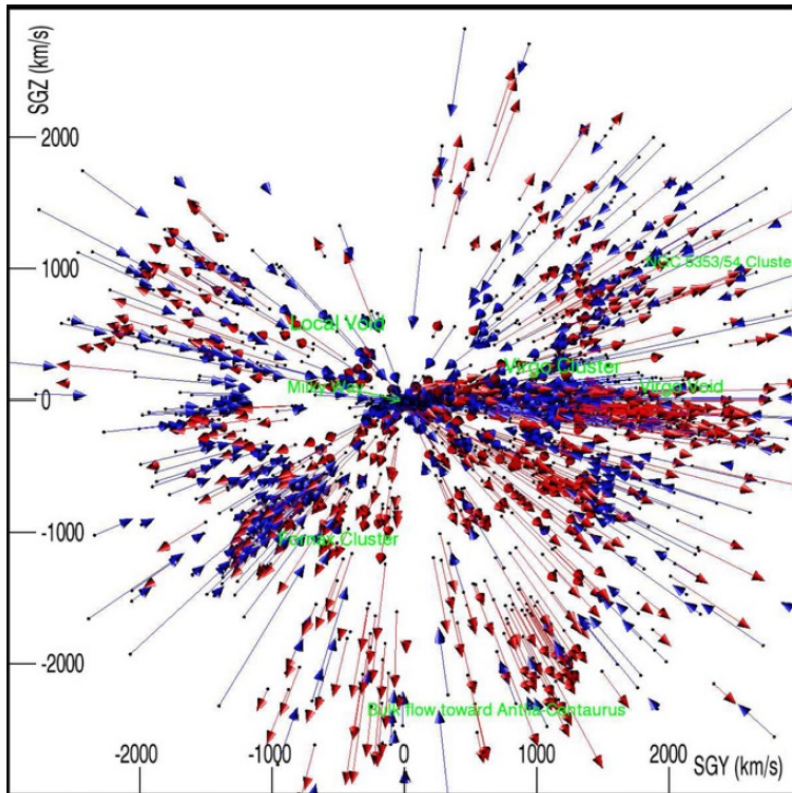
- PSCz map
- linear regime reconstruction
- Romano-Diaz & vdW 2007
Branchini et al. 1999

$$\mathbf{g}(\mathbf{r}, t) = -\frac{1}{a} \nabla \phi = \frac{3\Omega H^2}{8\pi} \int d\mathbf{x}' \delta(\mathbf{x}', t) \frac{(\mathbf{x}' - \mathbf{x})}{|\mathbf{x}' - \mathbf{x}|^3}$$



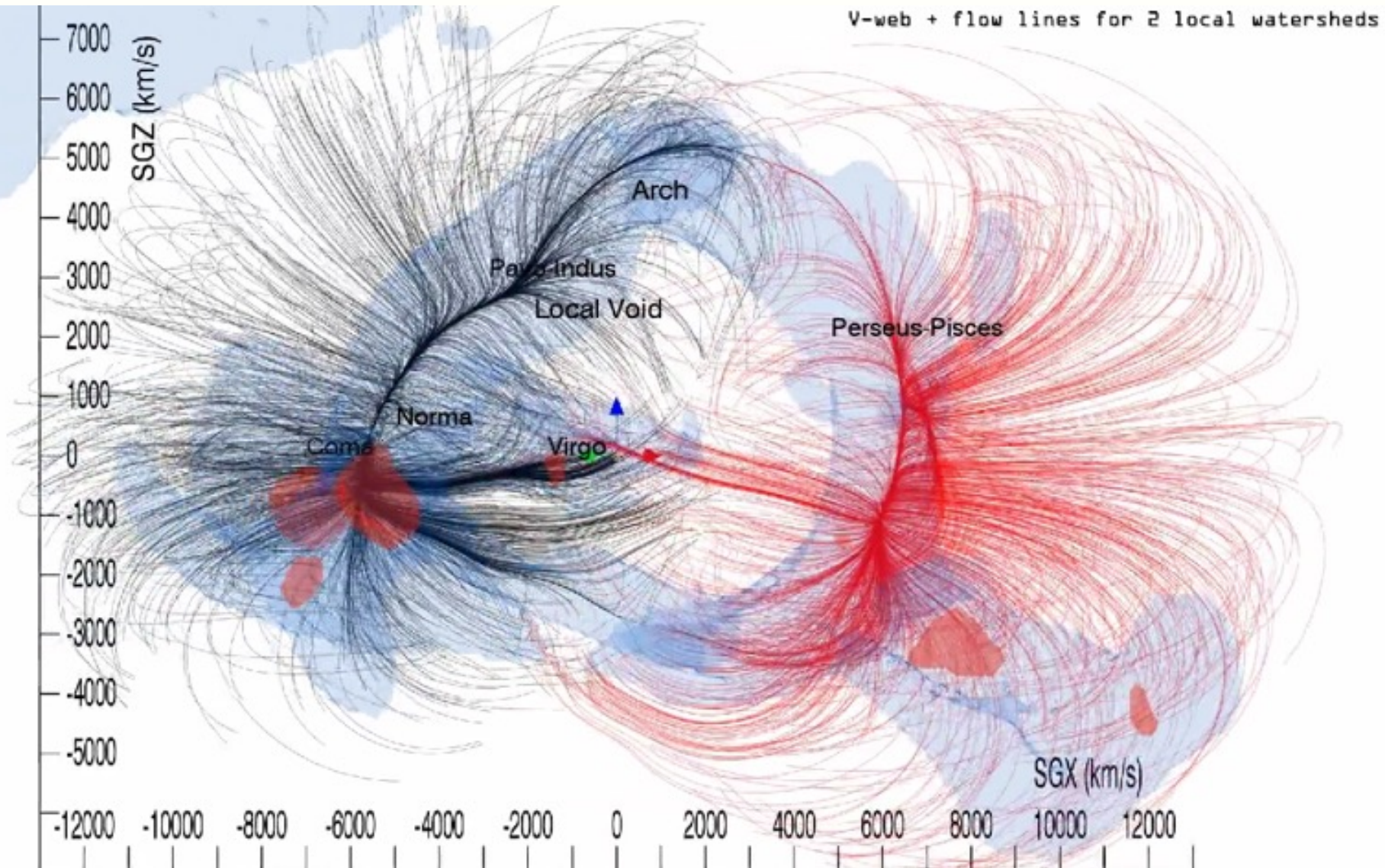
$$\mathbf{v}(\mathbf{x}, t) = \frac{H}{4\pi} \frac{f(\Omega_m)}{b} a \int d\mathbf{x}' \delta_{gal}(\mathbf{x}', t) \frac{(\mathbf{x}' - \mathbf{x})}{|\mathbf{x}' - \mathbf{x}|^3}$$

CosmicFlows-2

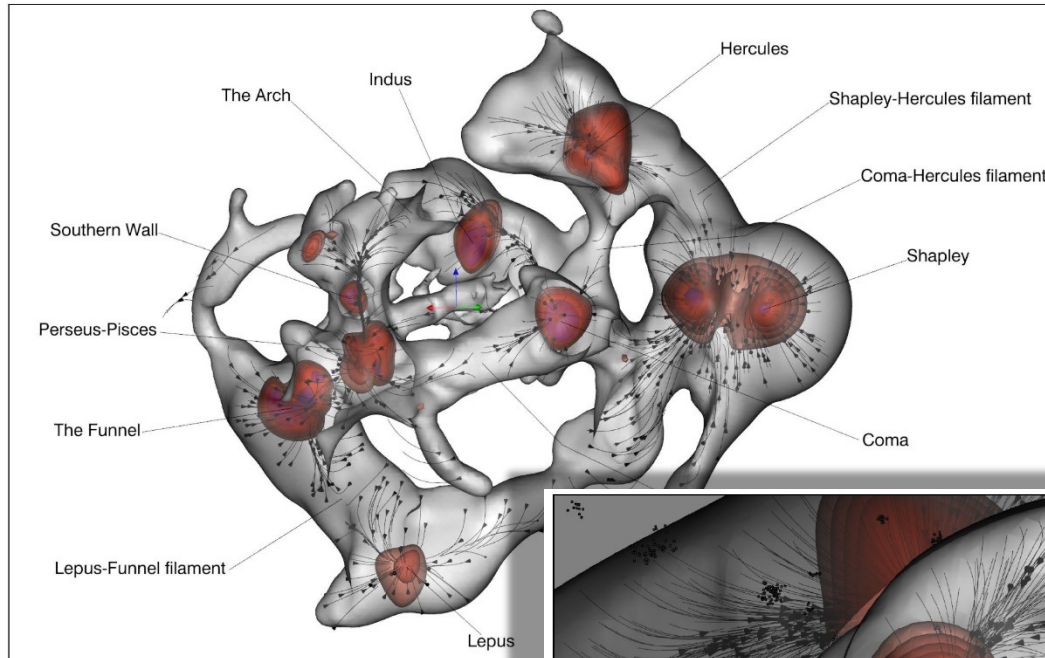


Courtois et al. 2013
Large scale migration flows in Cosmicflows-2

Laniakea

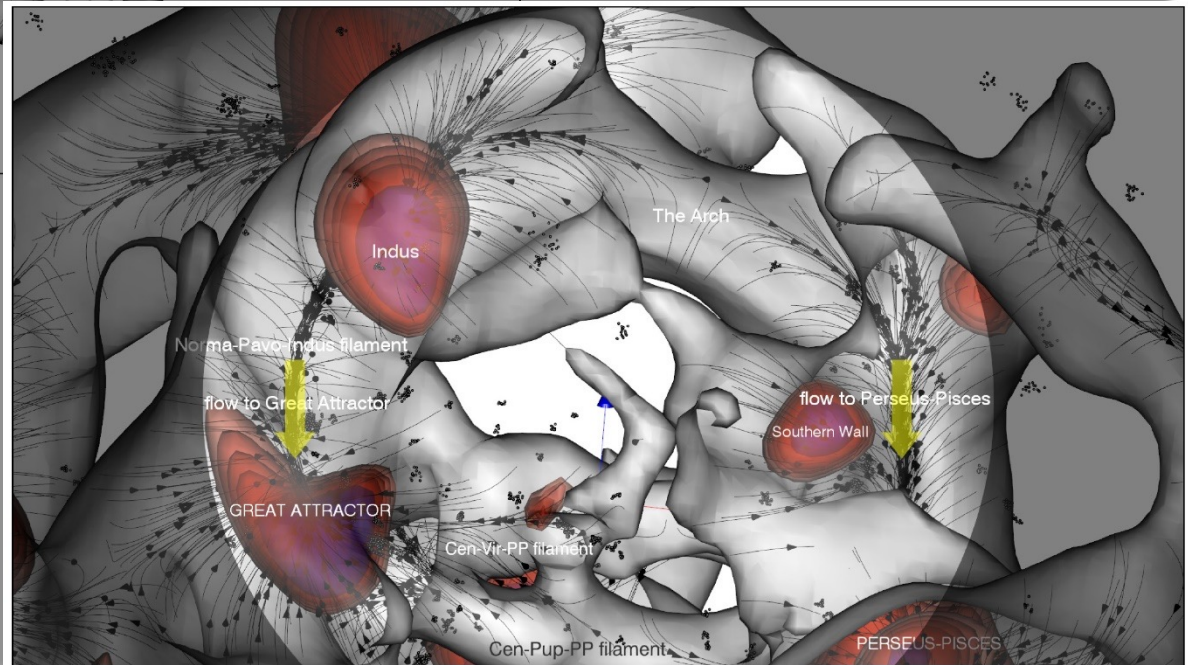


CosmicFlows-3



**Cosmic Web morphology:
velocity shear based
V-web identification
flow pattern in cosmic web**

(Pomarede et al. 2017)



Void Outflow

Void Formation

Void Evolution

an illustration

cosmology:

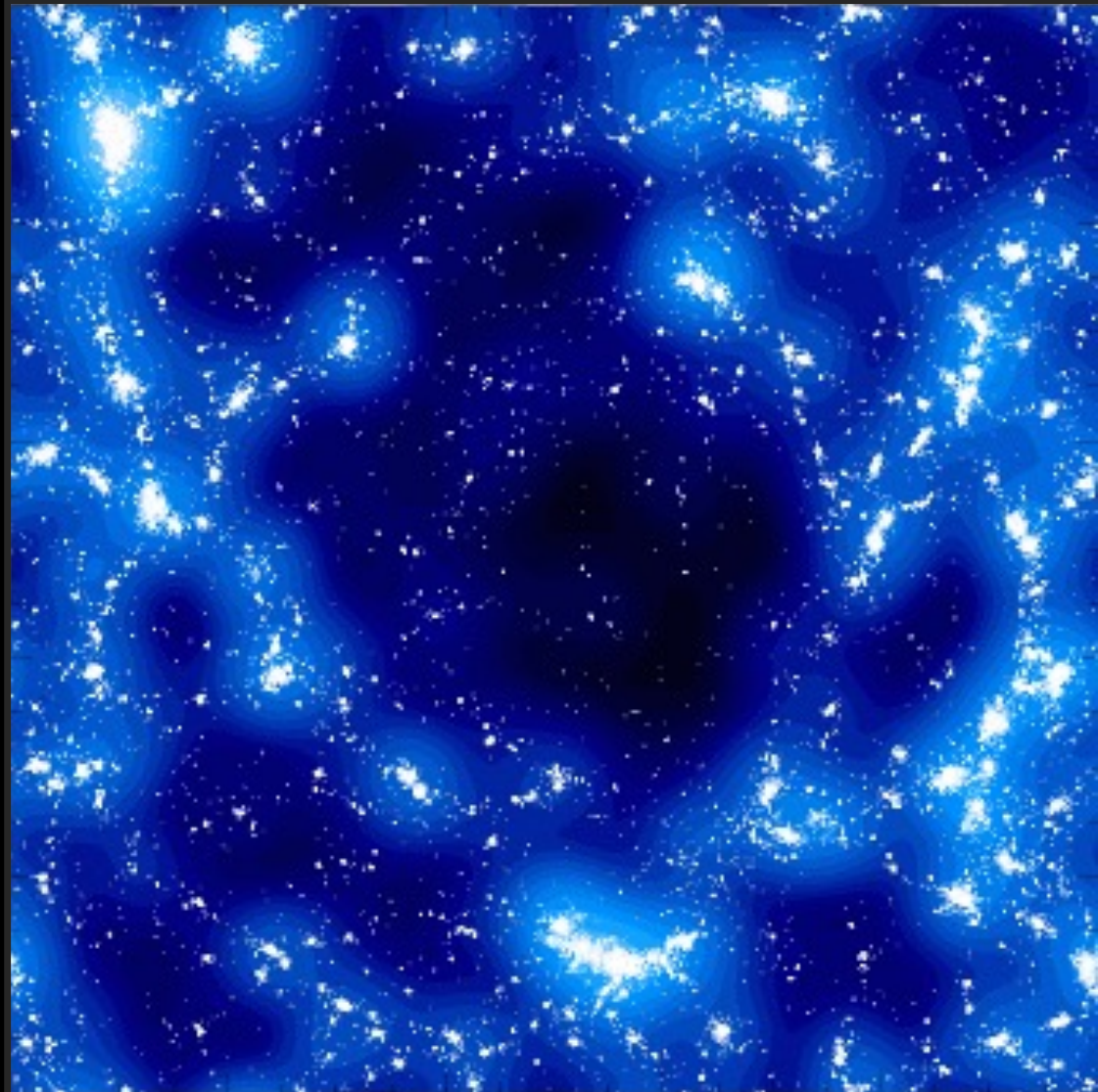
$$\Omega_m = 1.0; \quad H_0 = 70 \text{ km / s / Mpc}$$

initial conditions:

underdensity, Gaussian field

$$R_G \sim 4h^{-1} \text{ Mpc}$$

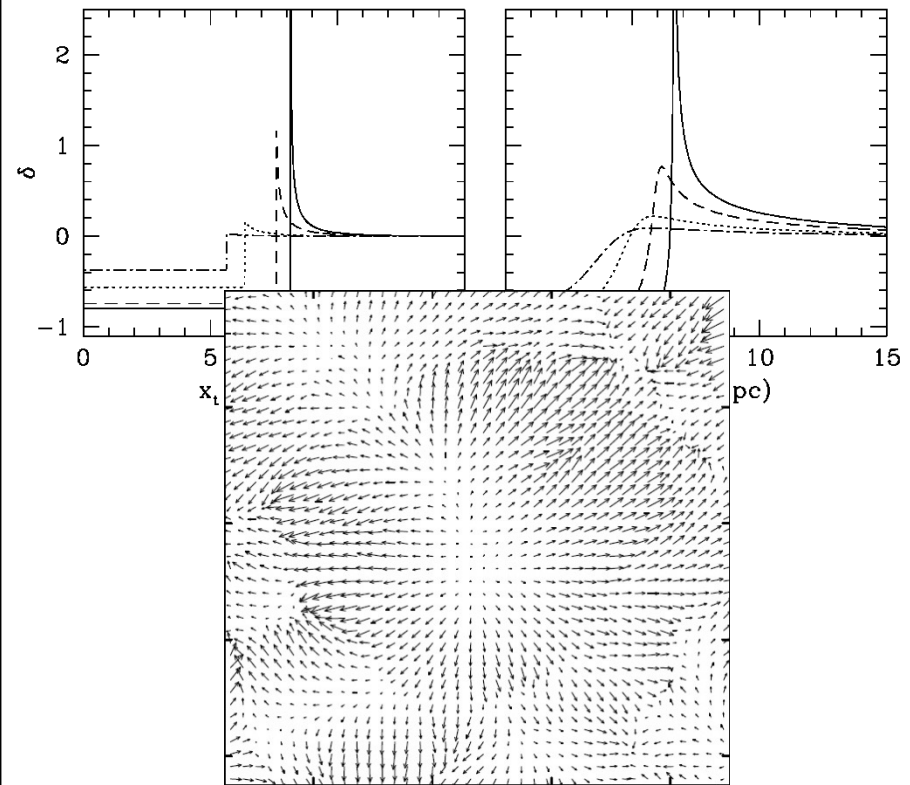
$$P(k) \propto k^{-0.5}$$



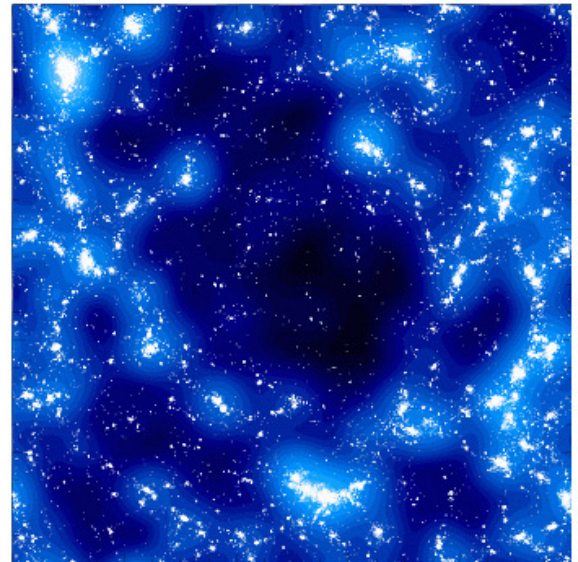
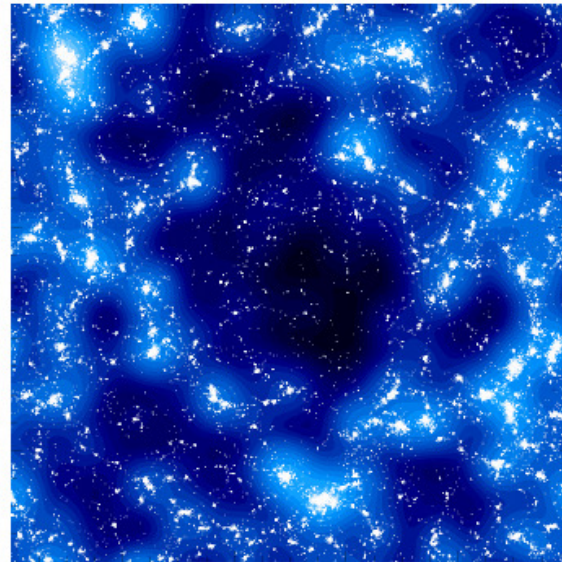
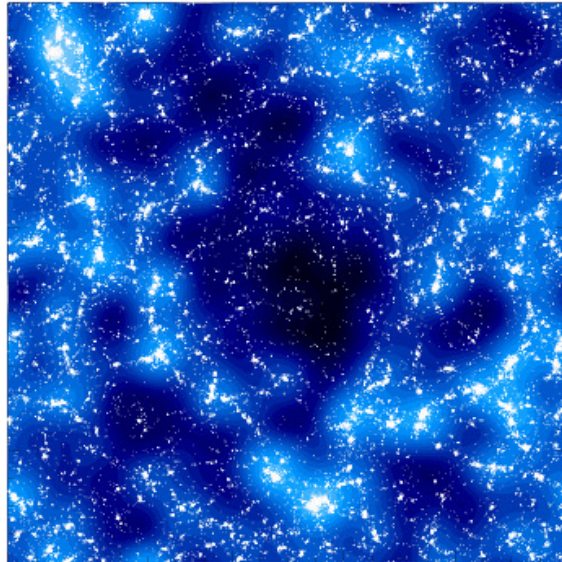
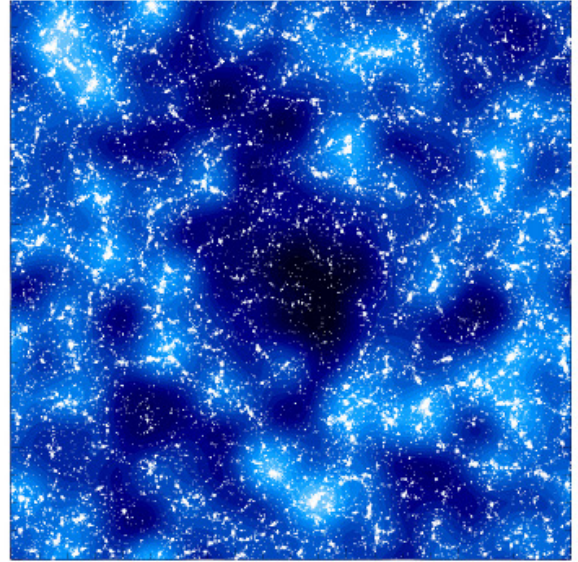
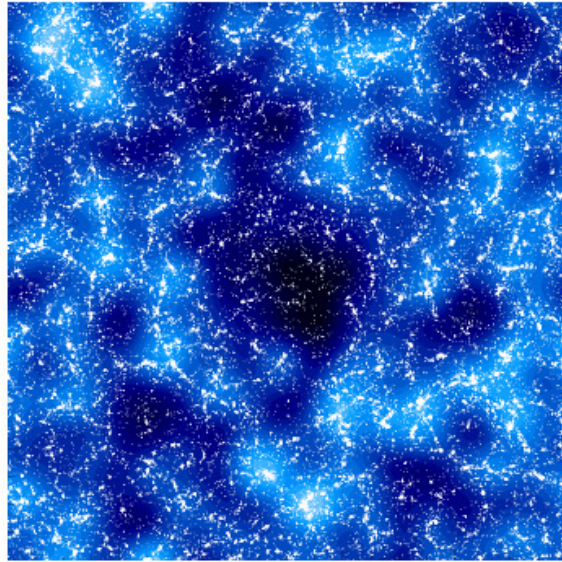
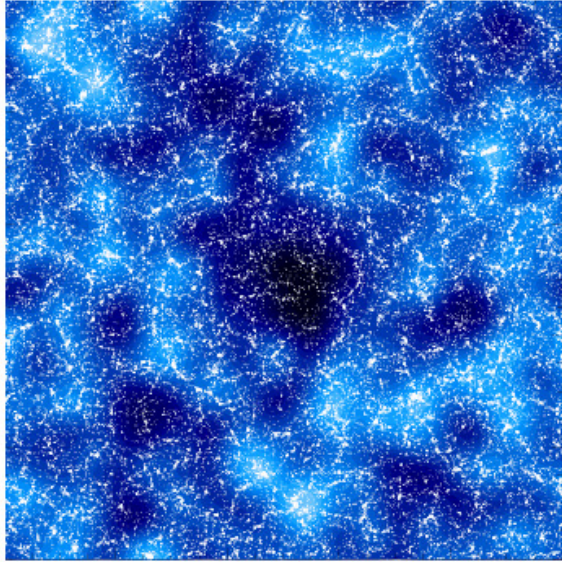
Void Superhubble Expansion

- **Superhubble Expansion**
tending towards “bucket” shape,
the void outflow is one with
uniform velocity divergence

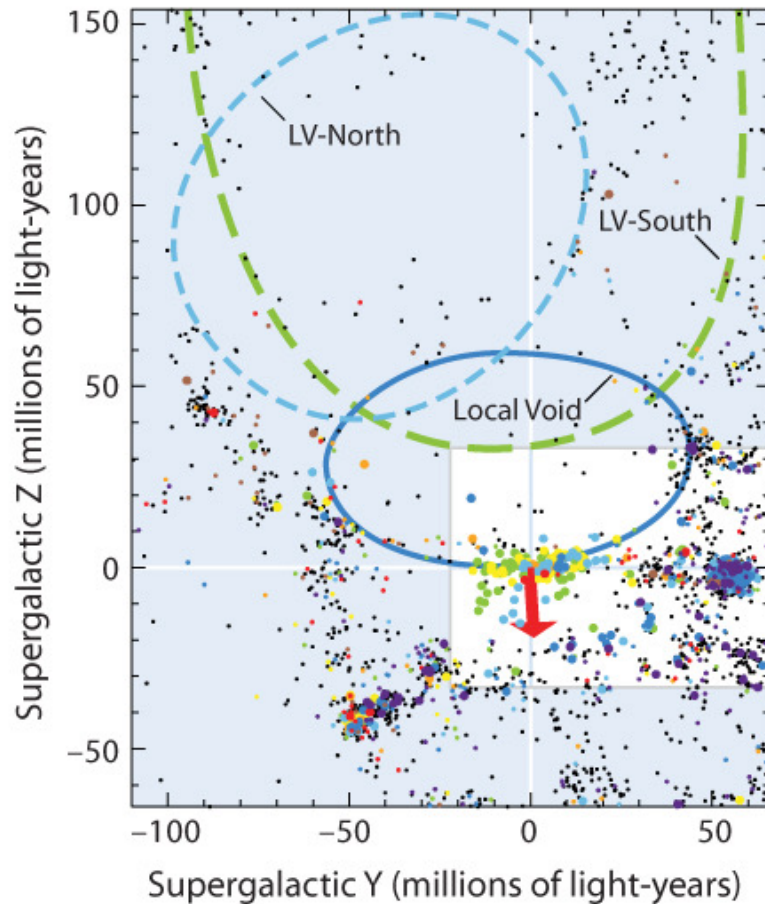
$$\theta = \frac{1}{H}(\nabla \cdot \vec{v}) \quad \Rightarrow \quad \theta_{\max} = 1.5\Omega^{0.6}$$



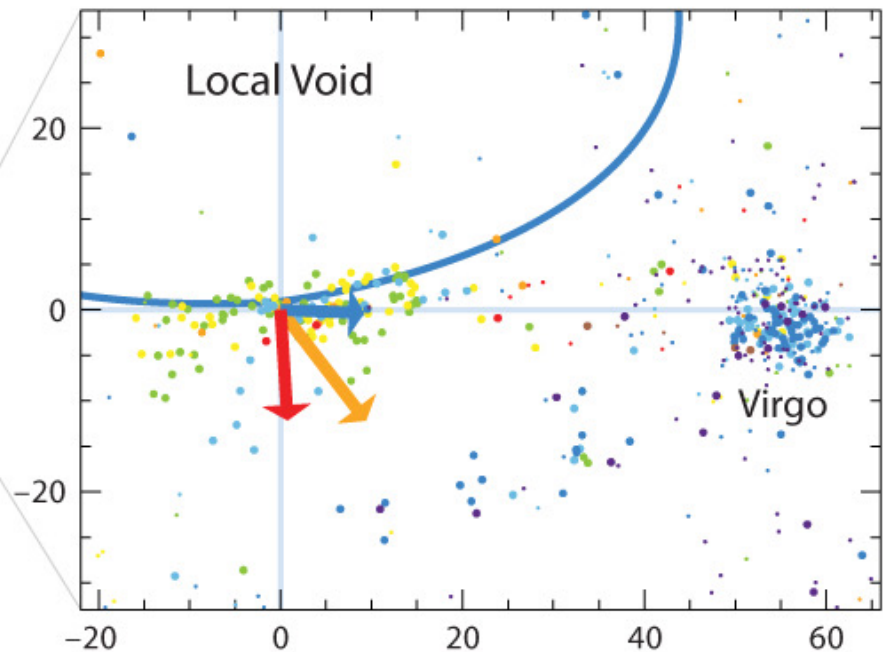
Void Formation



Push of the Local Void



Our motion with the respect to galaxies in the Local Supercluster *Tully et al. 2008, ApJ, 676, 184*



Tully et al. 2008:

Local Void pushes with ~ 260 km/s against our local neighbourhood

Galaxy Alignments

Spinning the Galaxies

Connection

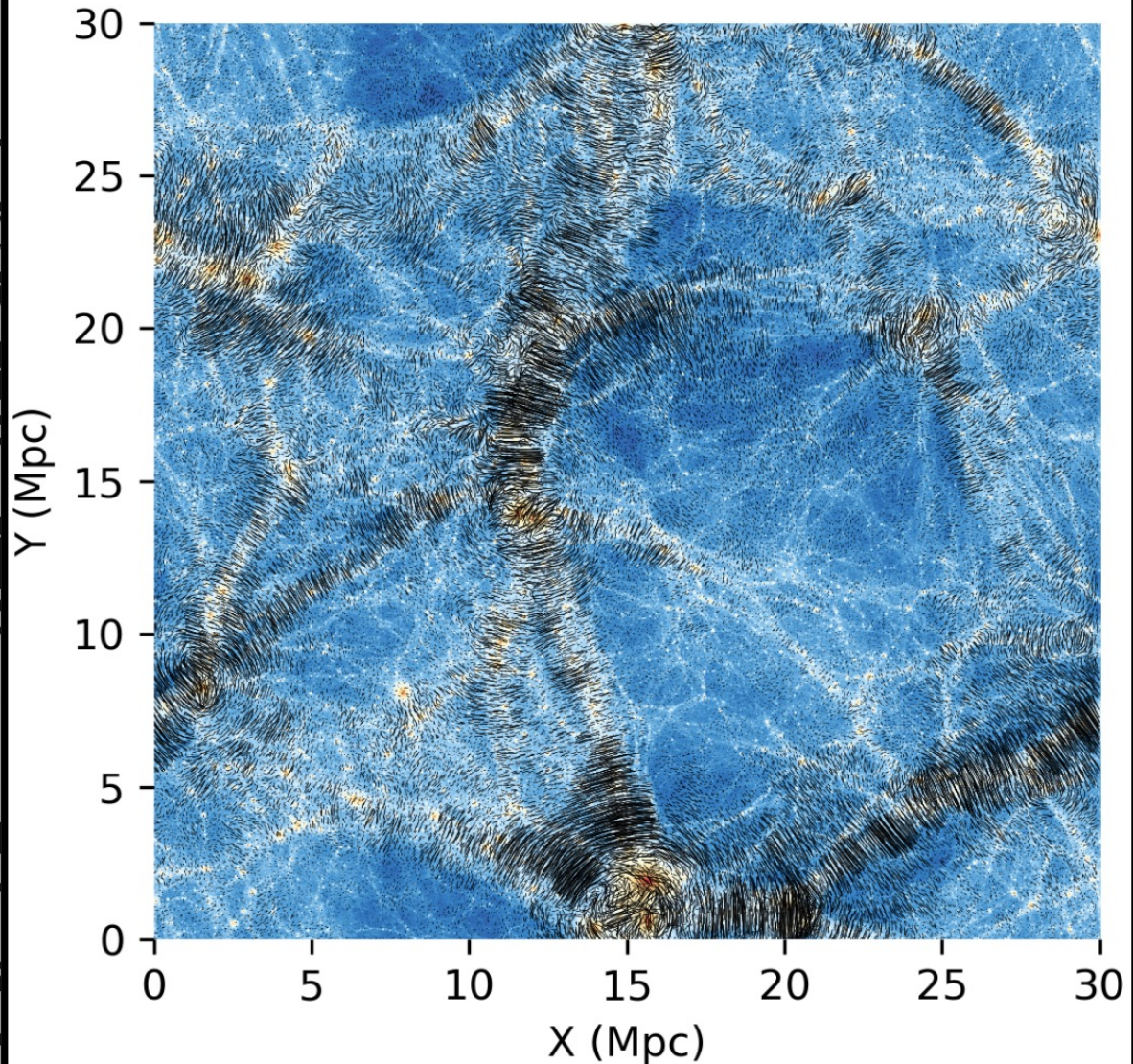
Galaxy Spin

Cosmic Web



Spinning the
Galaxies:

Tidal Forces that also
Shape Cosmic Web



Spinning the Galaxies



Angular Momentum Halo:

Collapsing halo gets tidally torqued into a rotating object:

Magnitude & Direction of resulting angular momentum vector L_i determined by

- protohalo inertia tensor I_{ij}
- driving tidal forces T_{ij}

$$L_i \propto \varepsilon_{ijk} T_{jm} I_{mk}$$

$\vec{L}$: angular momentum

T_{jm} : tidal tensor; I_{mk} : inertial tensor

ε_{ijk} : Levi – Civita symbol

Angular Momentum Alignment:

Look at correlation between angular momentum L_i
And generating large scale tidal force tensor T_{ij}

Lee & Pen (2000) parameterization to incorporate
correlation Tidal and Inertia tensor:

$$\langle L_i L_j | T \rangle \propto \frac{1}{3} \delta_{ij} + c \left(\frac{1}{3} \delta_{ij} - T_{ik} T_{kj} \right)$$

$c = 0$: *randomly distributed angular momentum*

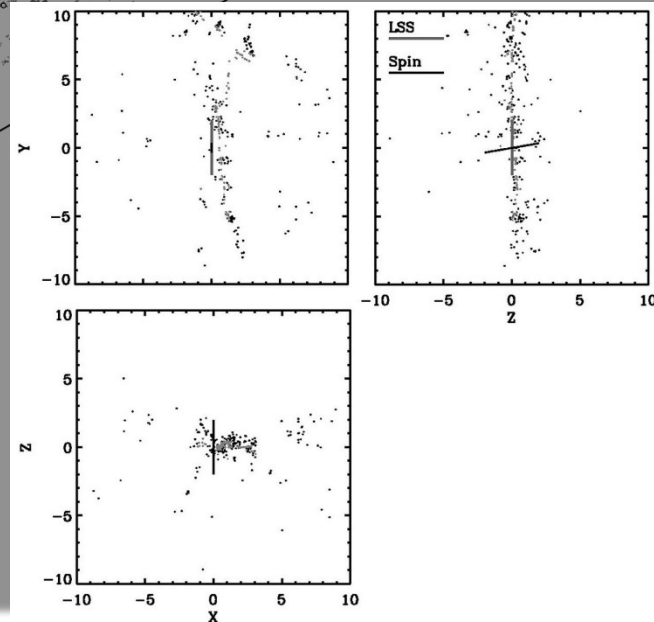
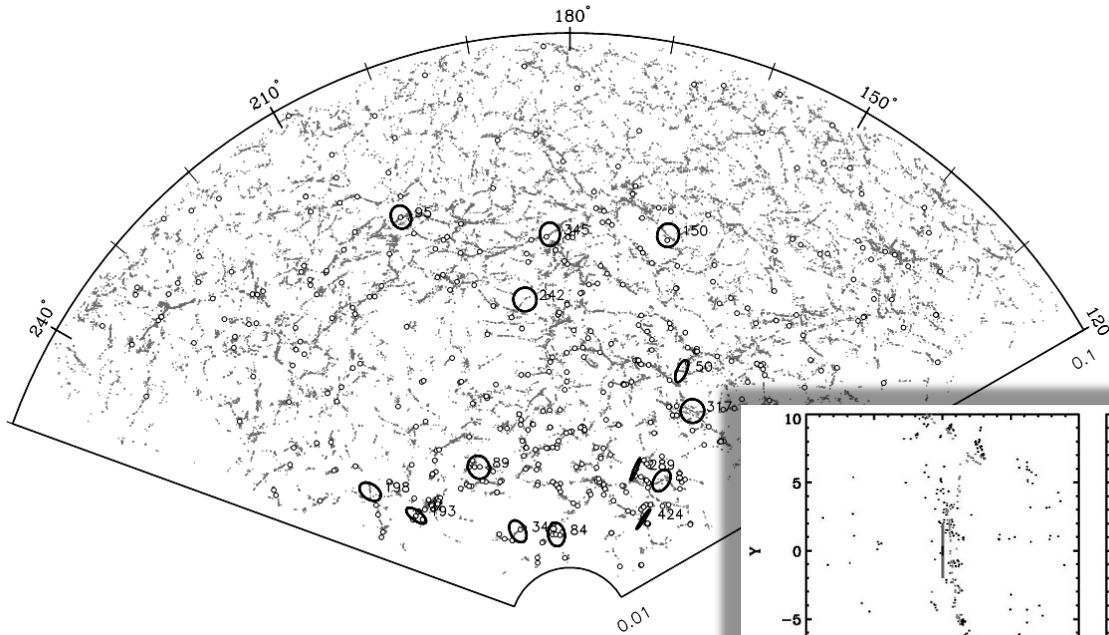
$c = 1$: *mutually dependent tidal and inertia tensors*

SDSS: Galaxy-Filament alignment

Aragon-Calvo et al. 2010

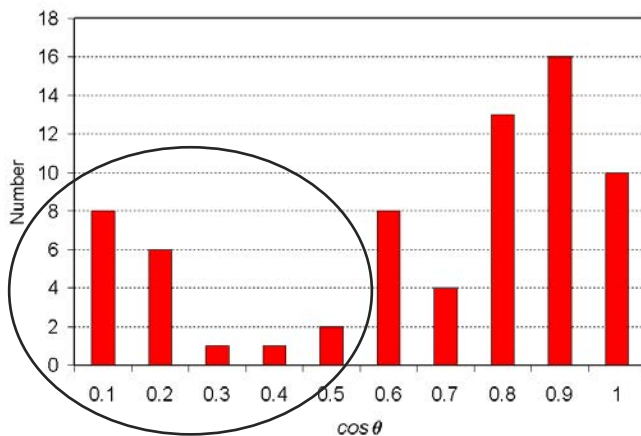
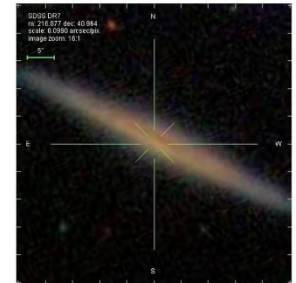
also see alignment studies by:

- Plionis et al. 1996
- Flin, Biernacka et al.
- Tempel & Libeskind 2014



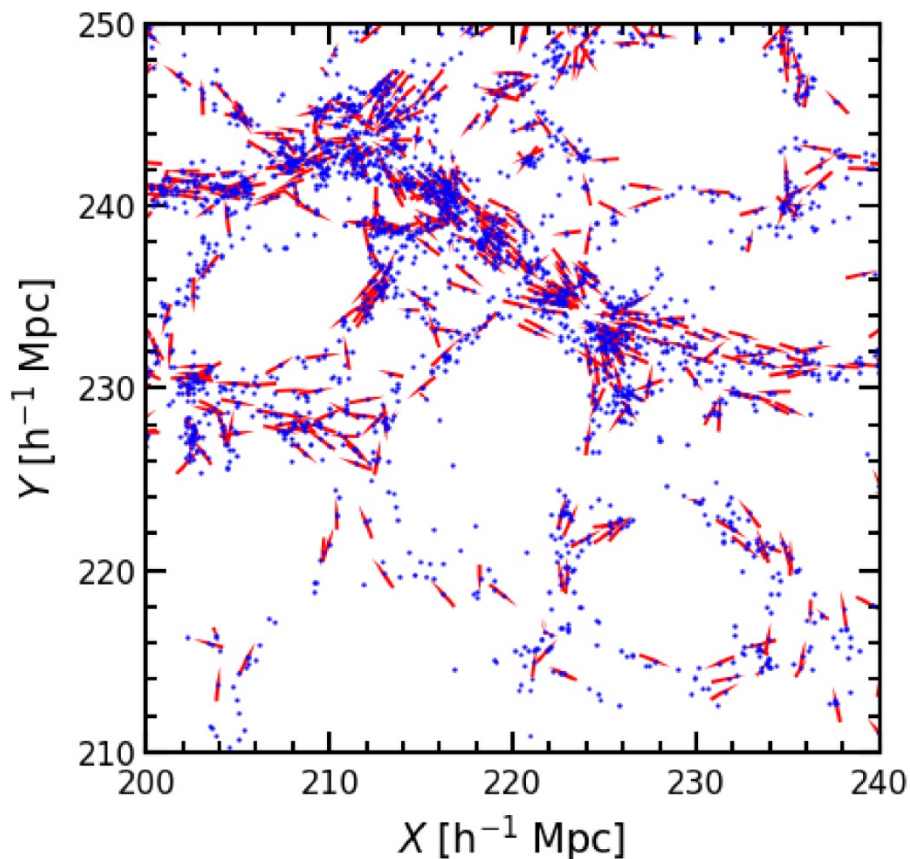
Gal 347

Spin-LSS = 0.169603
 $g-r = 0.988908$
 $M_r = -17.8010$
 $RA = 216.877$
 $DEC = 40.9637$
 $Z = 0.0183643$

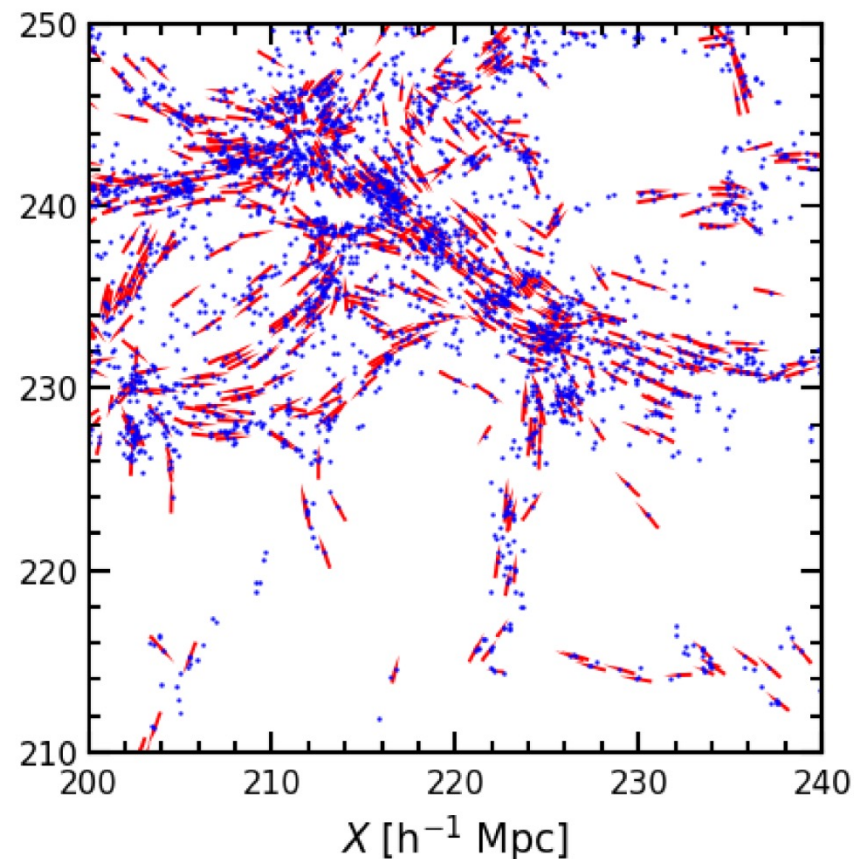


Filament-Halo alignments

(a) Haloes assigned to filaments by NEXUS+.



(b) Haloes assigned to filaments by NEXUS_velshear.



Spinning the Galaxies

Connection

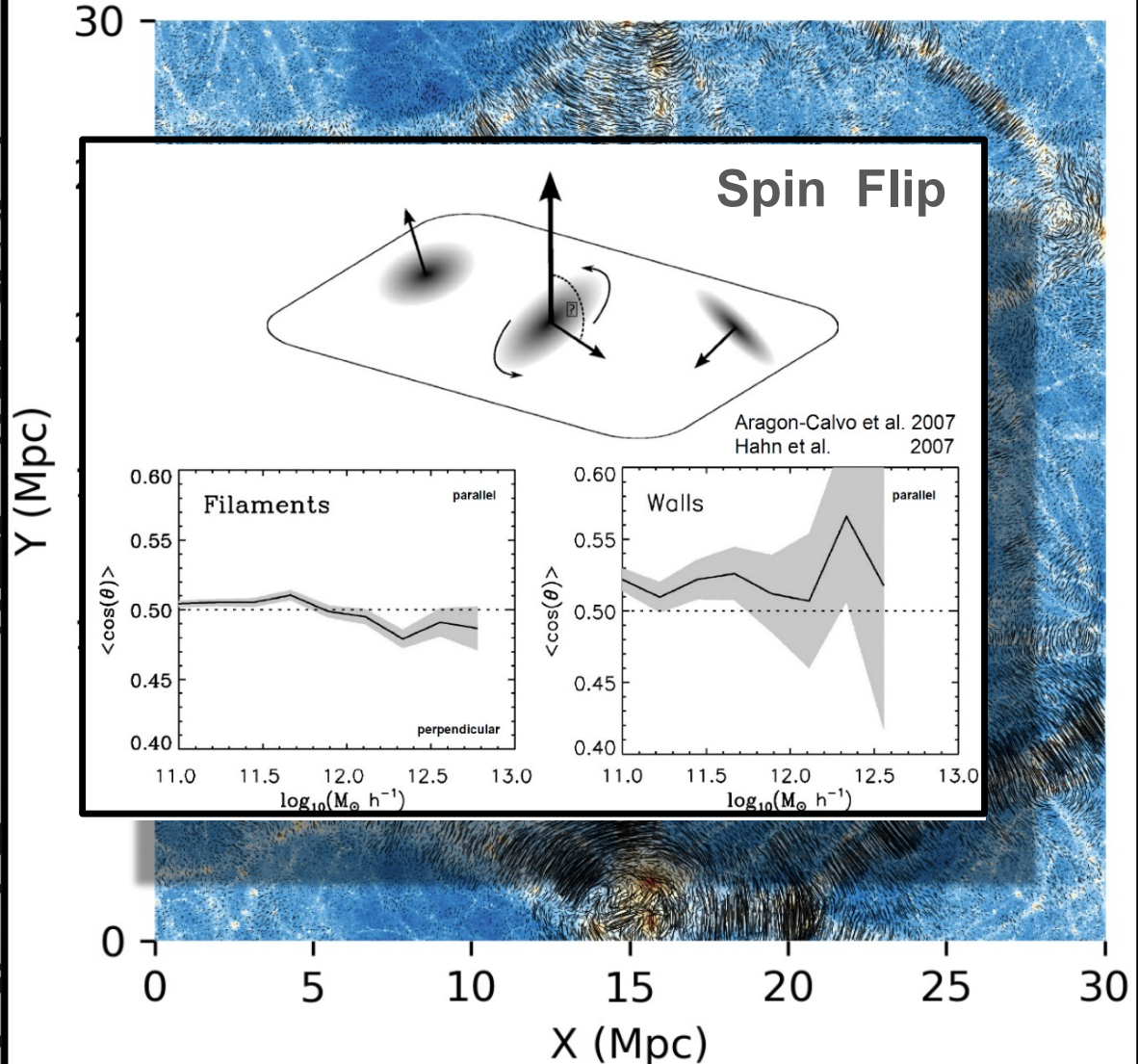
Galaxy Spin

Cosmic Web



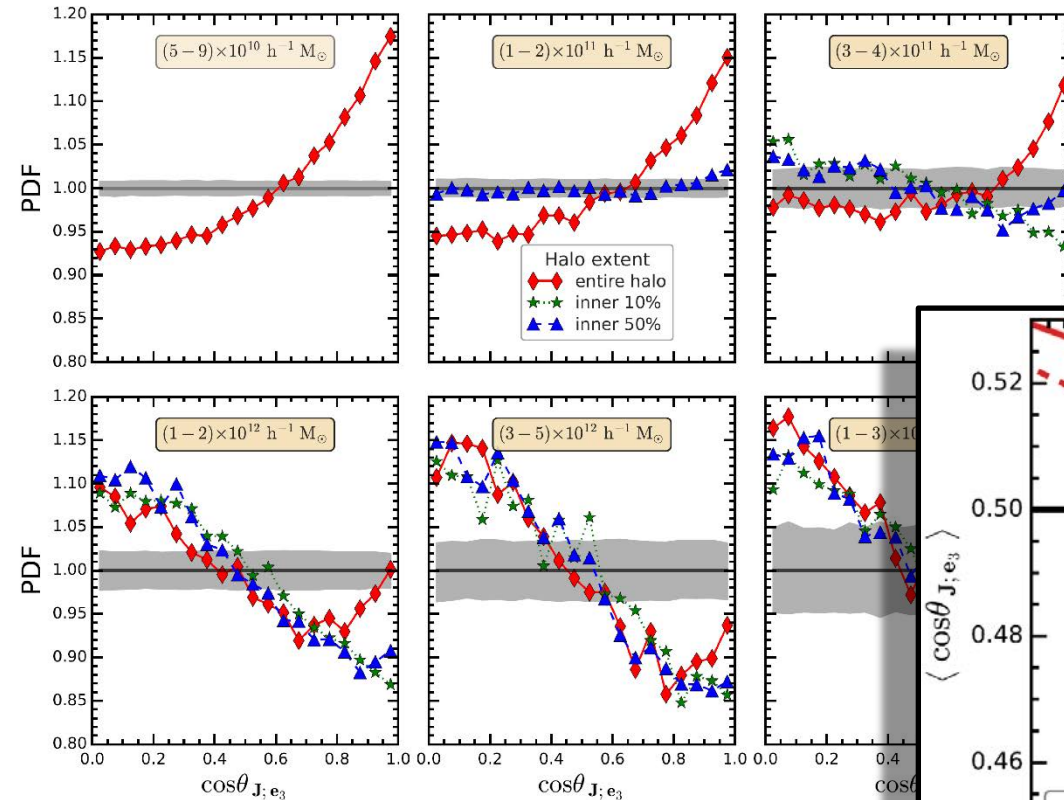
Spinning the
Galaxies:

Tidal Forces that also
Shape Cosmic Web

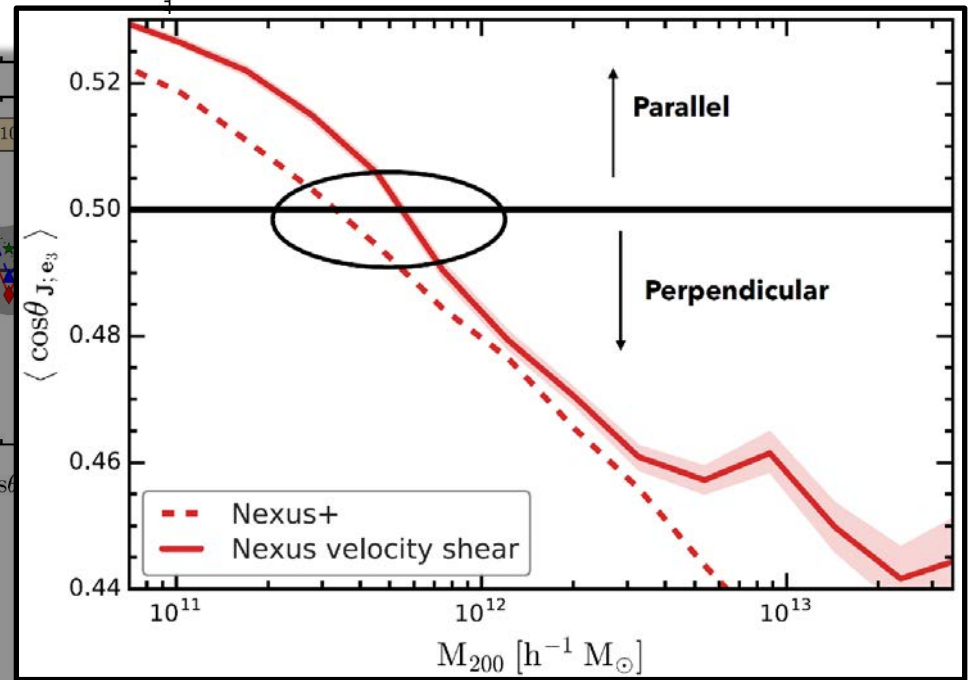


Spin-Flip: halo mass dependent alignments

Ganeshaiah et al. 2018, 2019, 2021

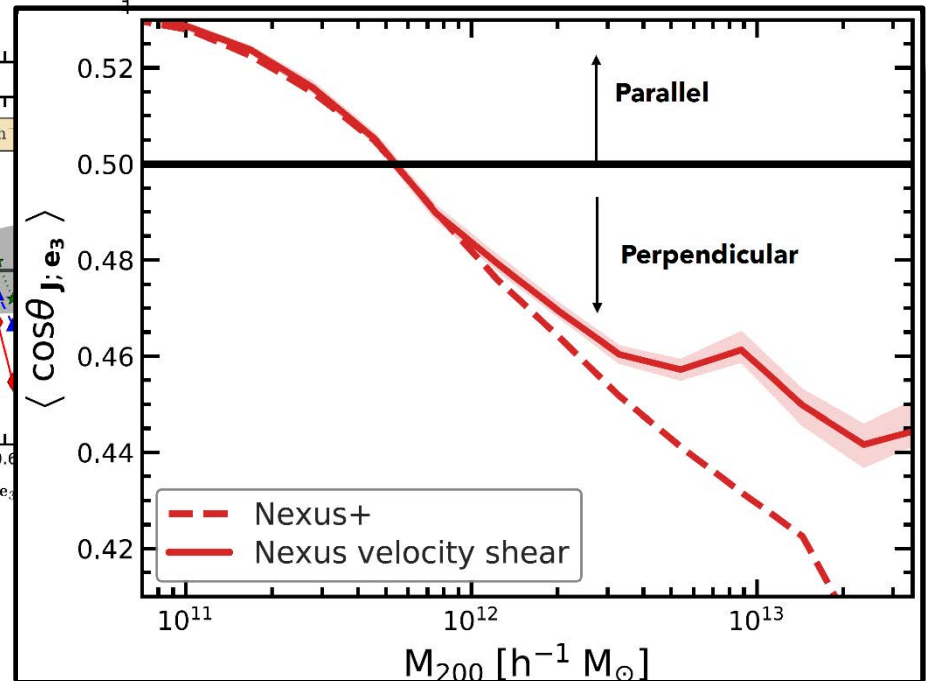
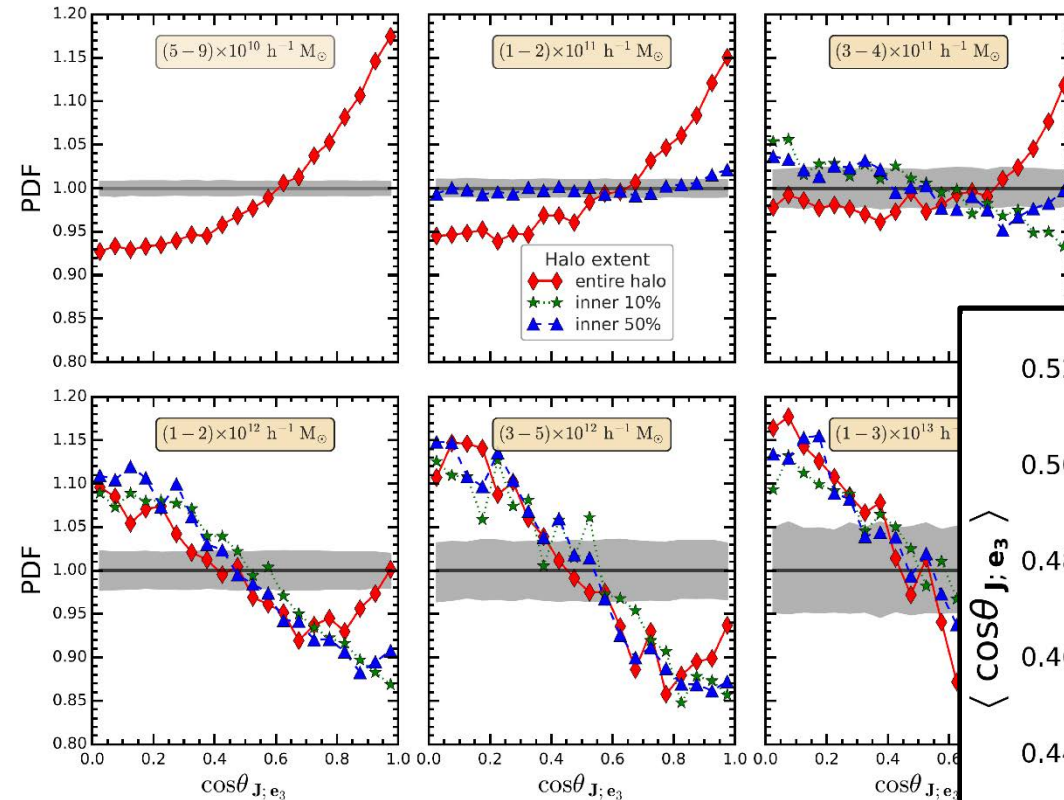


Aragon-Calvo, vdW, Jones & van der Hulst 2007
 Hahn et al. 2007,
 Codis et al. 2012, Welker et al. 2020, Kraljic et al. 2019,
 Ganeshaiah et al. 2018, 2019, 2021, Paz et al. 2021



Spin-Flip: filament identity

Ganeshaiah et al. 2018, 2019, 2021



Aragon-Calvo, vdW, Jones & van der Hulst 2007

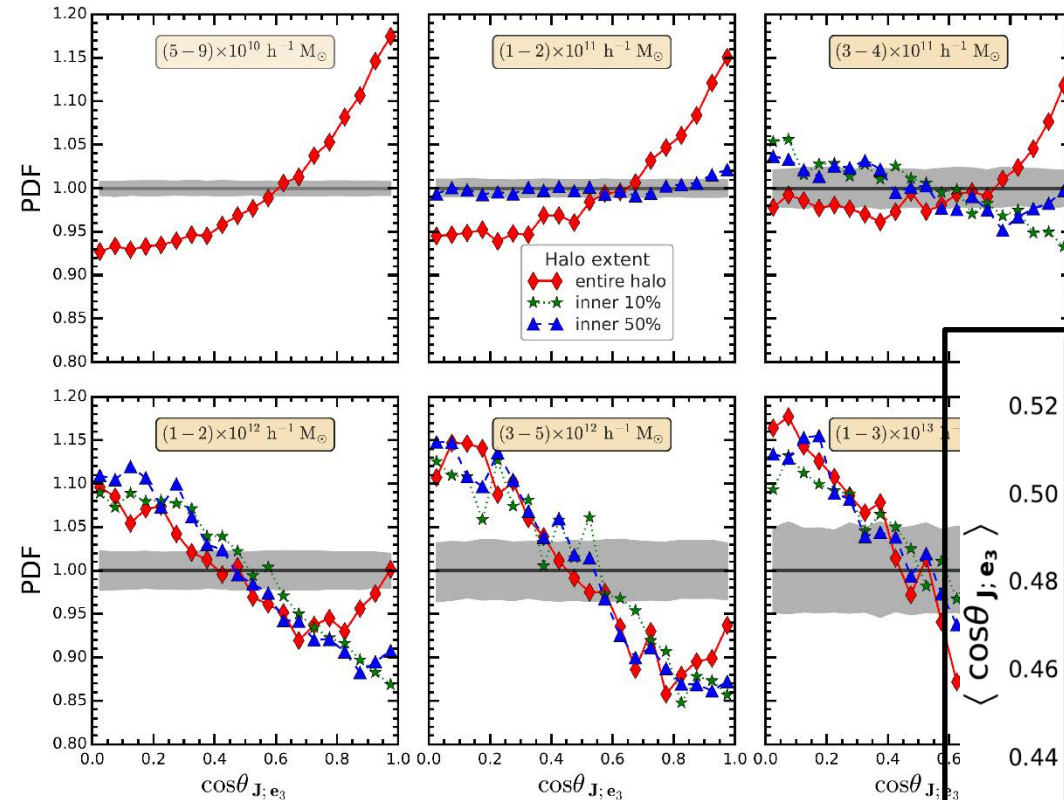
Hahn et al. 2007,

Codis et al. 2012, Welker et al. 2020, Kraljic et al. 2019,

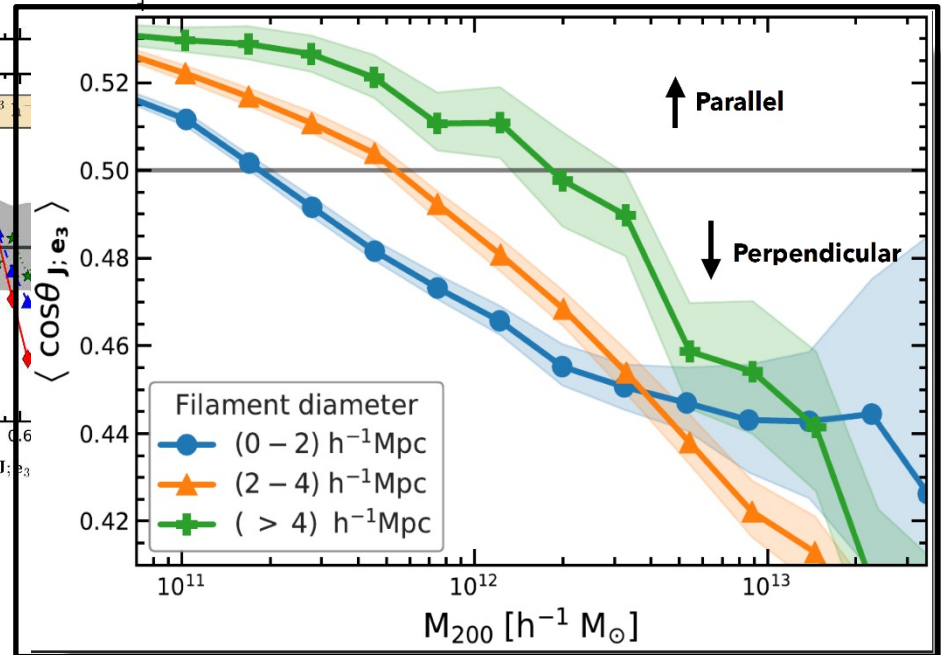
Ganeshaiah et al. 2018, 2019, 2021, Paz et al. 2021

Spin-Flip: filament thickness

Ganeshaiah et al. 2018, 2019, 2021

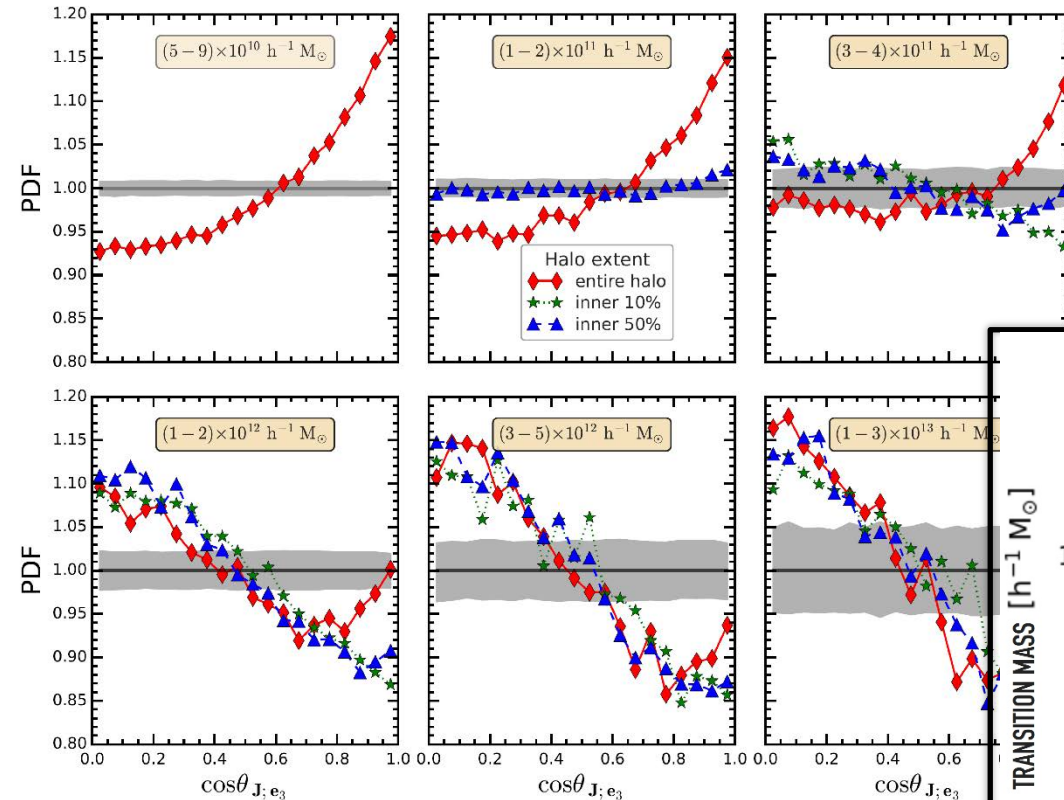


Aragon-Calvo, vdW, Jones & van der Hulst 2007
 Hahn et al. 2007,
 Codis et al. 2012, Welker et al. 2020, Kraljic et al. 2019,
 Ganeshaiah et al. 2018, 2019, 2021, Paz et al. 2021



Spin-Flip: filament thickness

Ganeshaiah et al. 2018, 2019, 2021

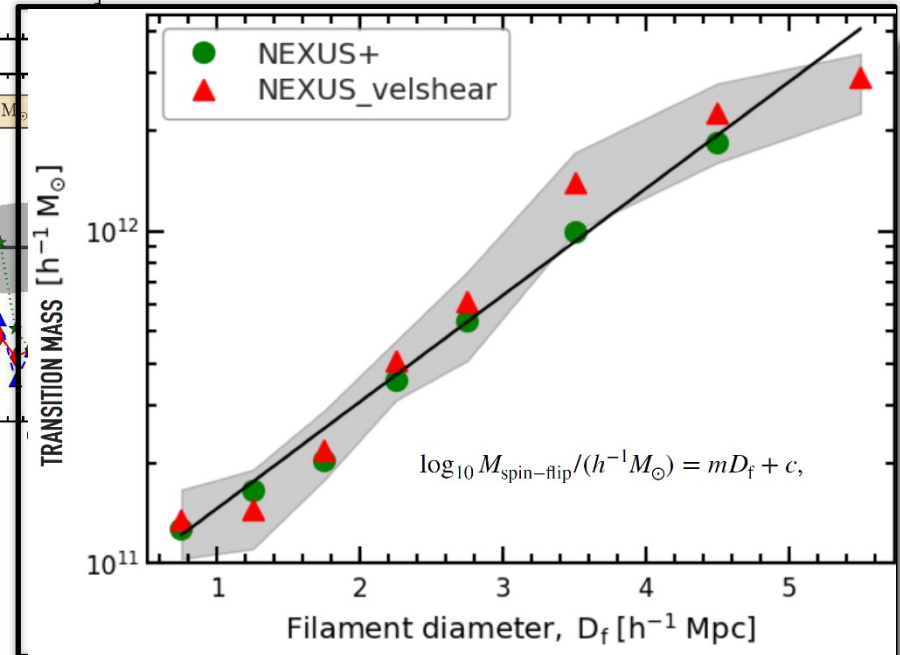


Aragon-Calvo, vdW, Jones & van der Hulst 2007

Hahn et al. 2007,

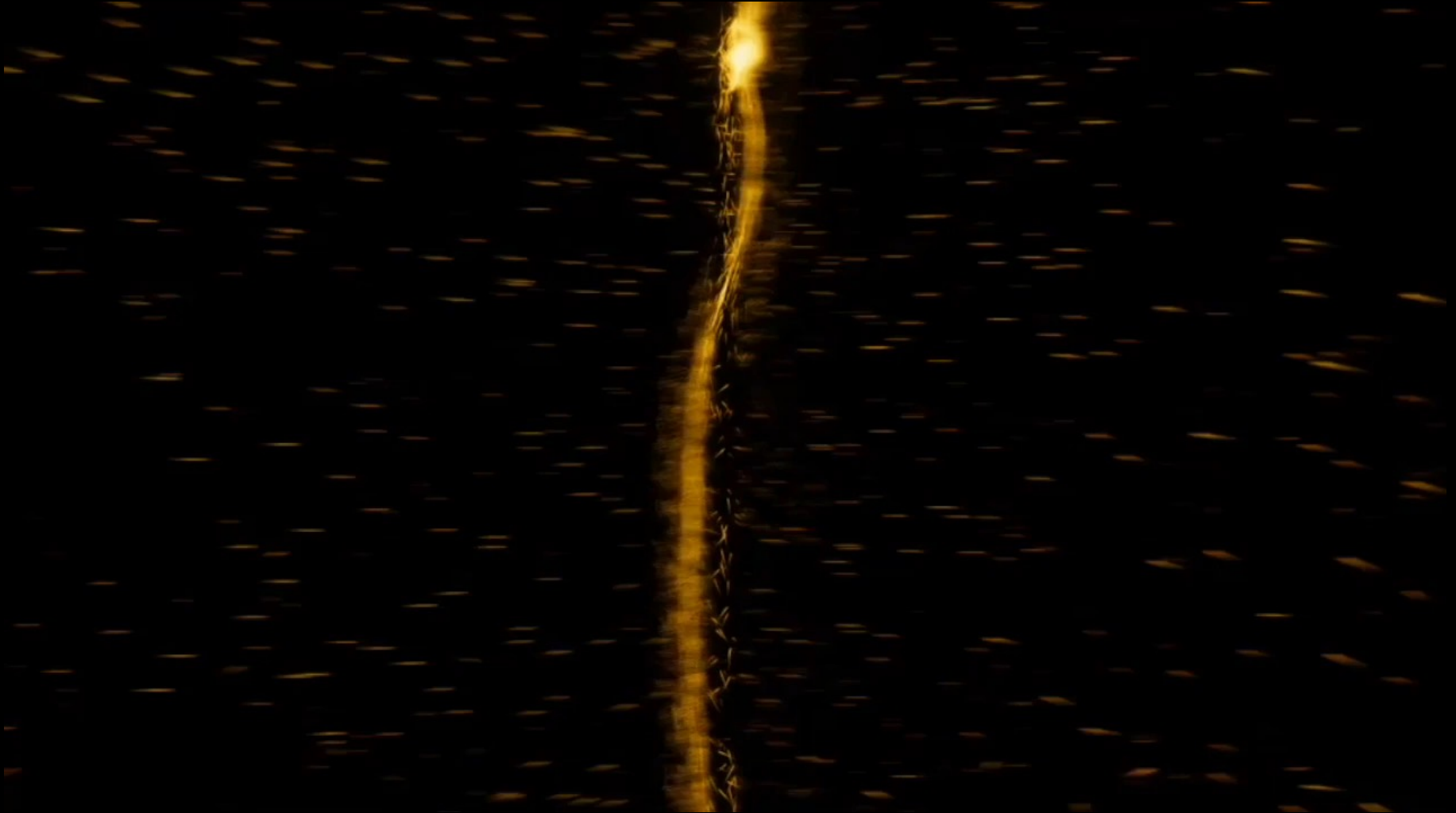
Codis et al. 2012, Welker et al. 2020, Kraljic et al. 2019,

Ganeshaiah et al. 2018, 2019, 2021, Paz et al. 2021

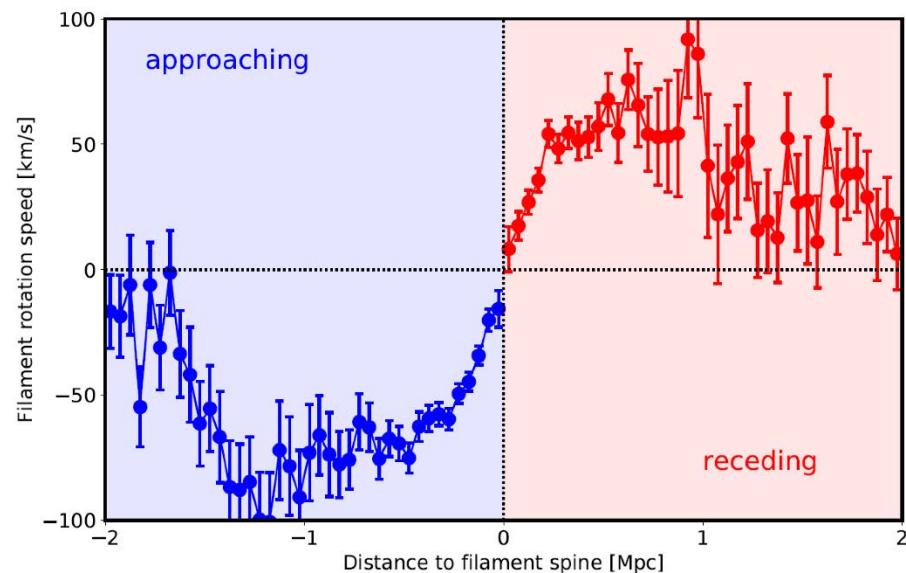
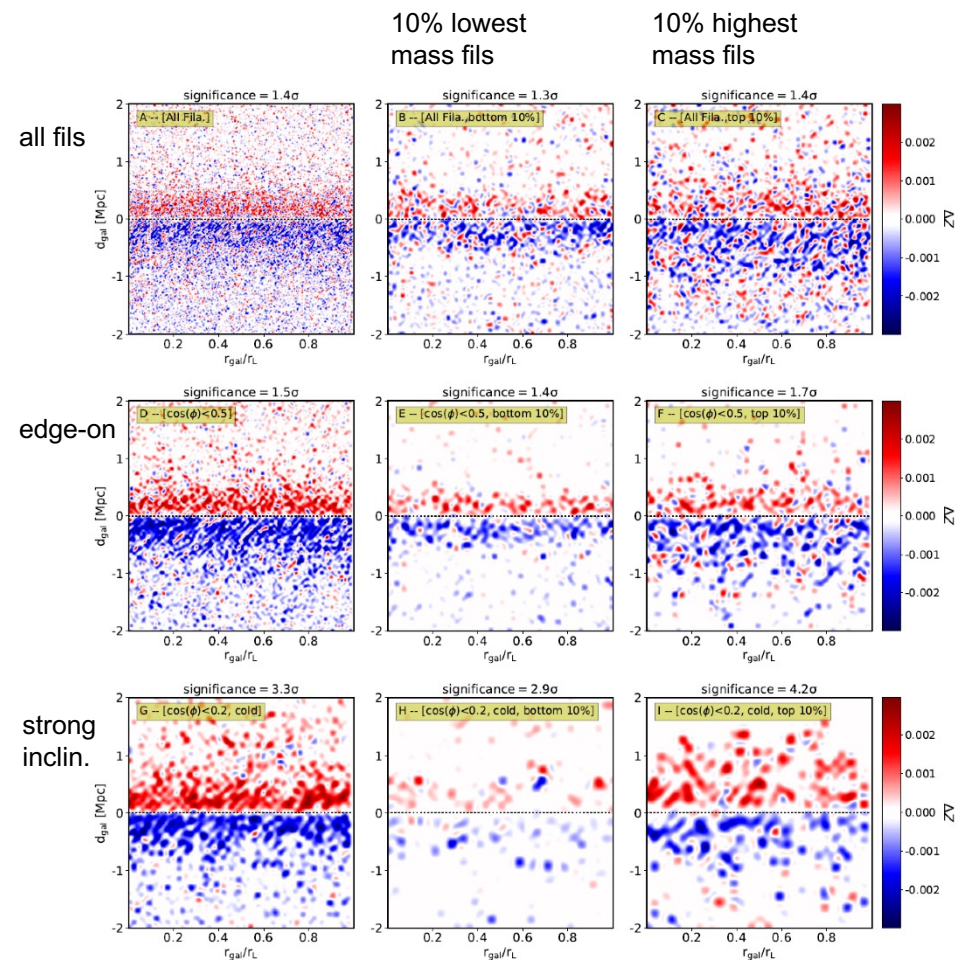


Filament Rotation

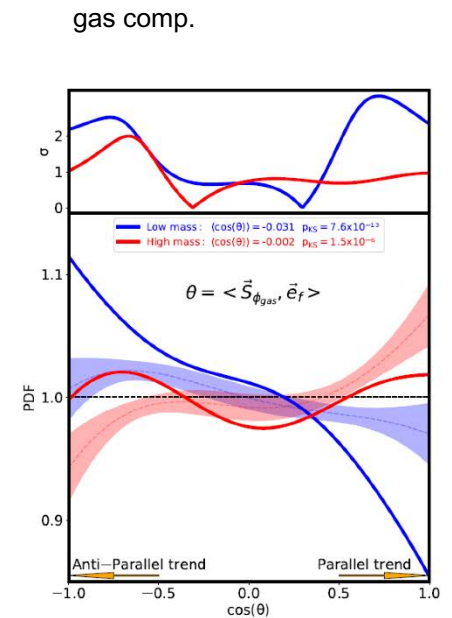
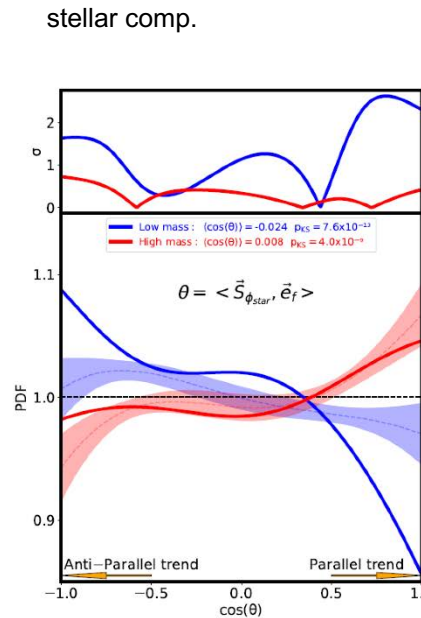
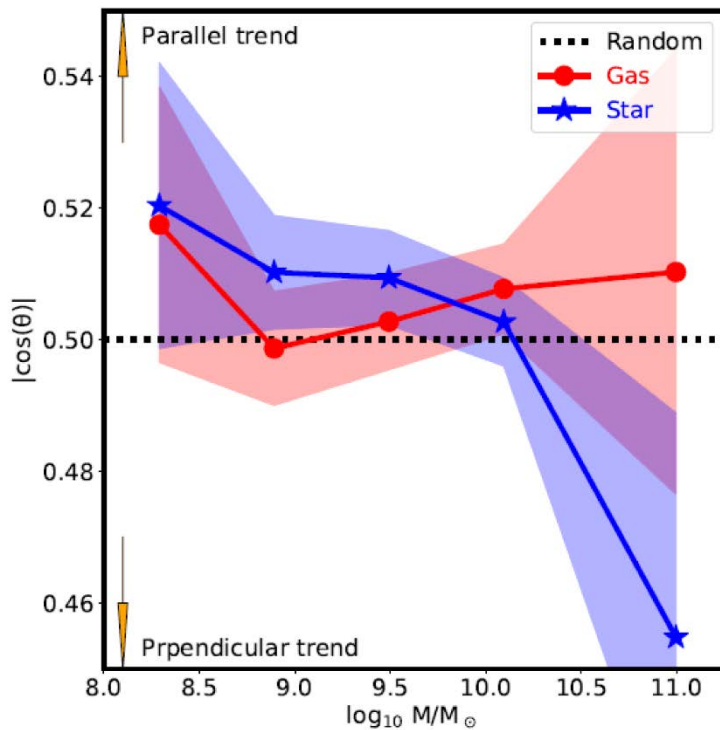
Spinning Filaments



Spinning Filaments



Spinning Filaments – Spinning Gals: parallel vs. anti-parallel ?



Wang et al. 2025

Sample MaNGA galaxies (IFU)

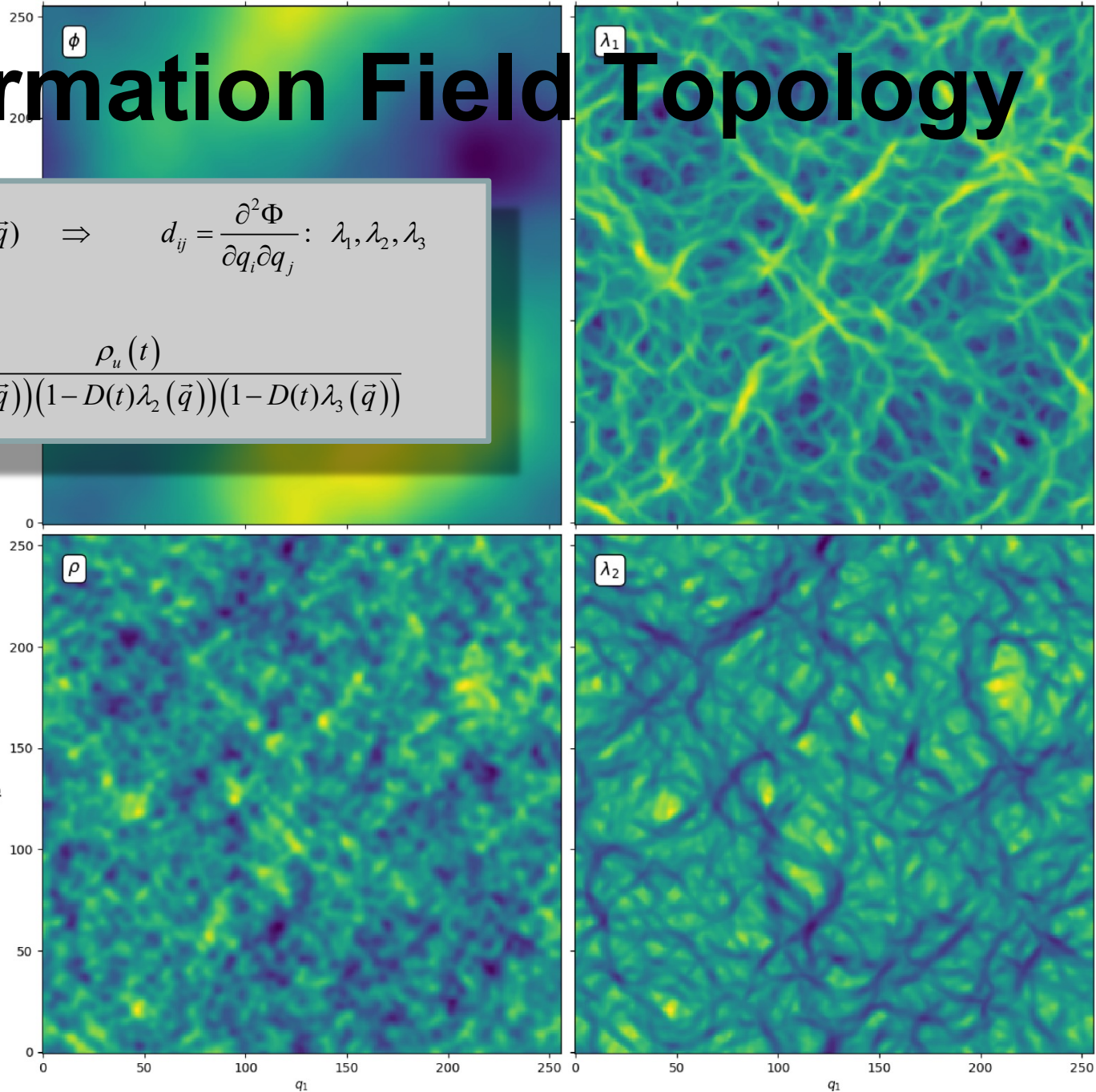
**Cosmic Web
Connectivity:**

**weaving the
Cosmic Tapestry**

Deformation Field Topology

$$\vec{x}(\vec{q}, t) = \vec{q} - D(t) \vec{\nabla} \Phi(\vec{q}) \quad \Rightarrow \quad d_{ij} = \frac{\partial^2 \Phi}{\partial q_i \partial q_j} : \lambda_1, \lambda_2, \lambda_3$$

$$\rho(\vec{q}, t) = \frac{\rho_u(t)}{(1 - D(t)\lambda_1(\vec{q}))(1 - D(t)\lambda_2(\vec{q}))(1 - D(t)\lambda_3(\vec{q}))}$$



**Deformation eigenvalue
Landscape**
(initial Gaussian field)

**Connectivity
Cosmic Web:**

**Topology
deformation field**

Formative agent of the Cosmic Web:

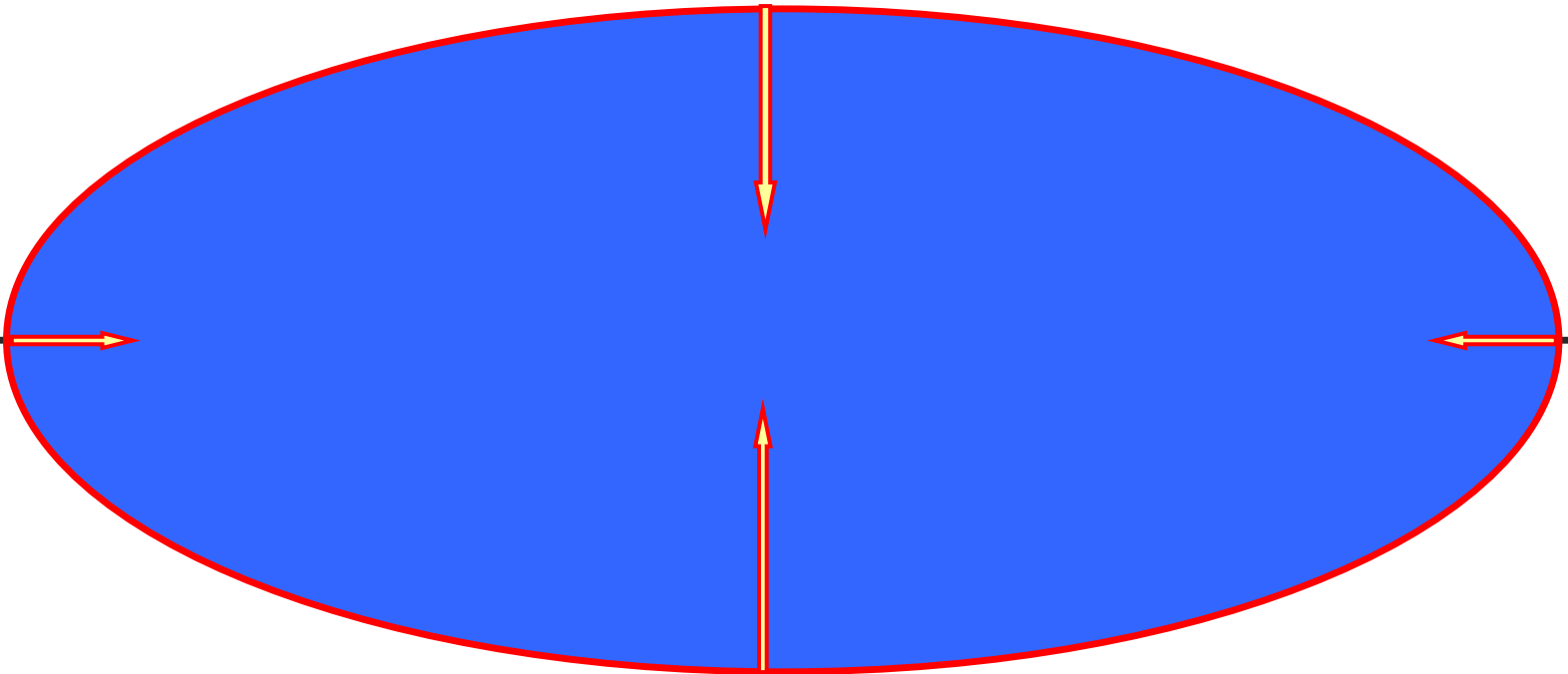
Tidal strain induced by the Megaparsec Matter Distribution:

- anisotropic collapse of structures
- connection clusters-filaments:
clusters main agent for stretching filaments

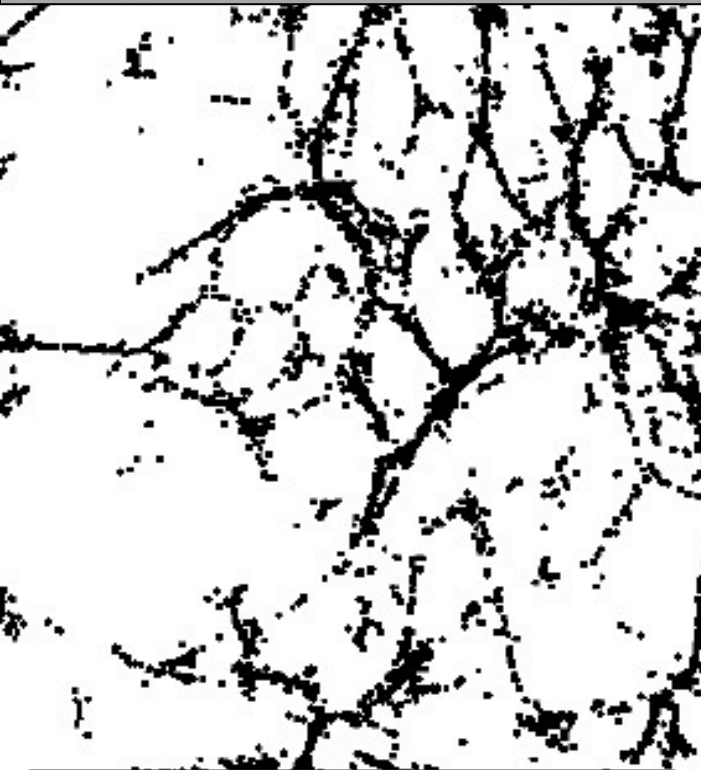
$$T_{ij}(\vec{r}, t) = \frac{3\Omega H^2}{8\pi} \int d\vec{x} \, \delta(\vec{x}, t) \left\{ \frac{3(x_i - r_i)(x_j - r_j) - |\vec{x} - \vec{r}|^2 \delta_{ij}}{|\vec{x} - \vec{r}|^5} \right\} - \frac{1}{2} \Omega H^2 \delta(\vec{r}, t) \delta_{ij}$$

Anisotropic Gravitational Collapse

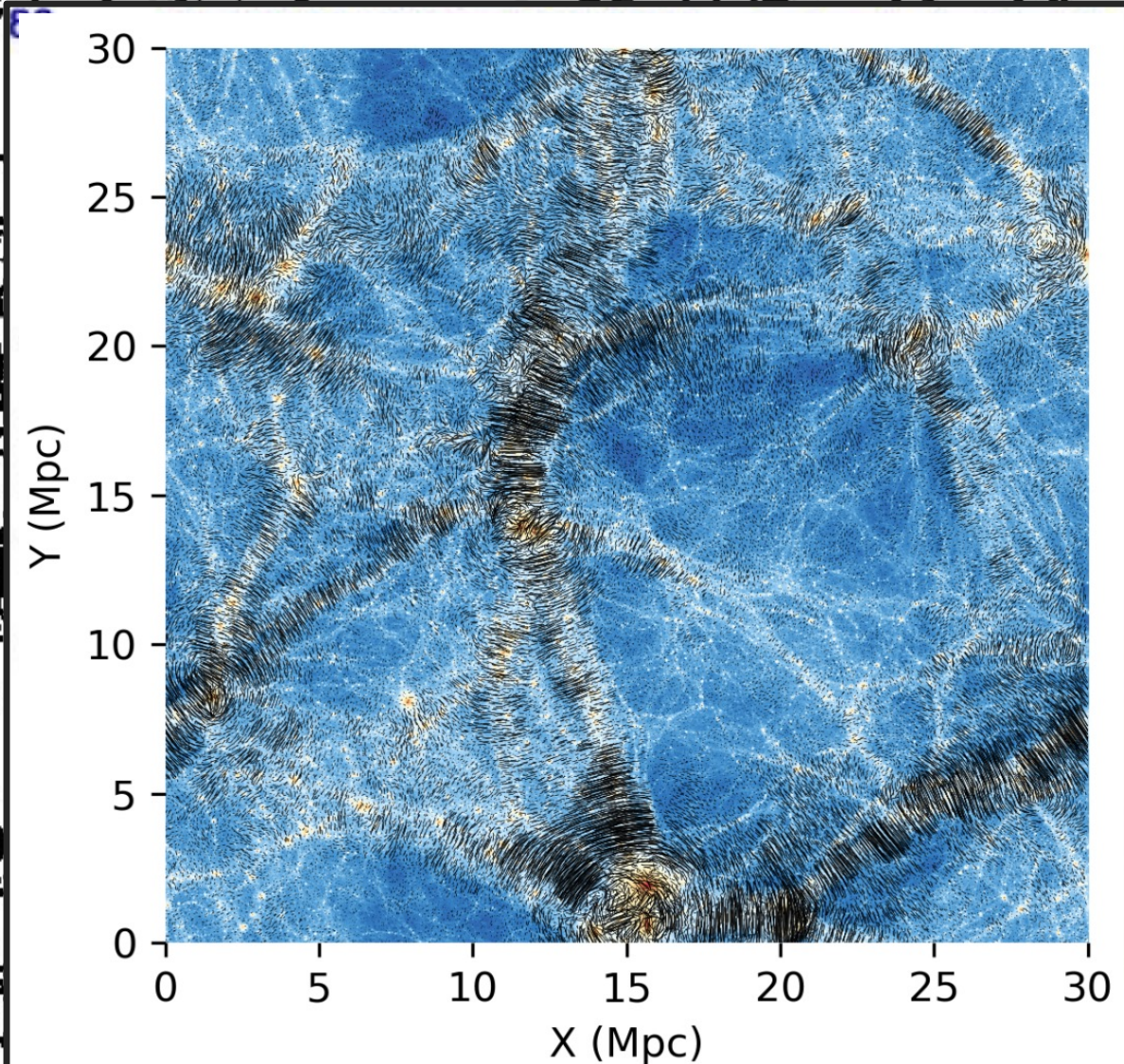
Amplification
small perturbations in gravity along different directions (tidal forces)



Tidal Shaping of the Cosmic Web



**Tidal Forces
shape the Cosmic Web**

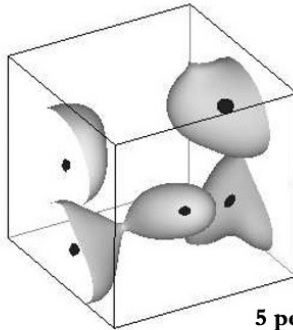


Tidal Shaping of the Cosmic Web

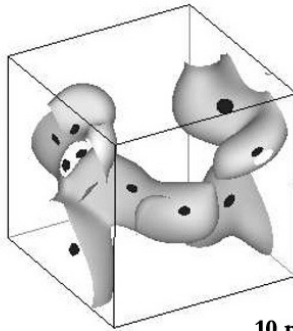
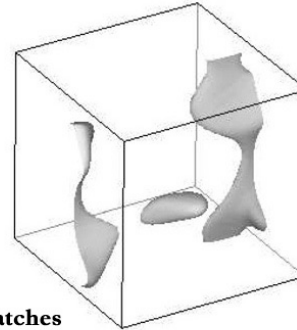
Cosmic Web Theory

Bond, Kofman &
Pogosyan 1996

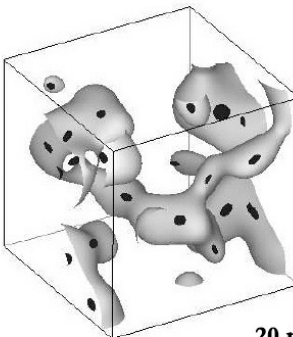
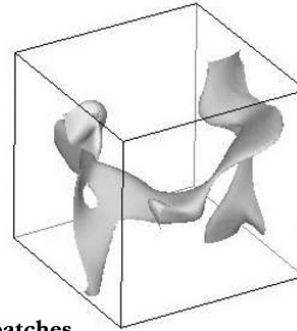
Tidal Forces:
main source are the
clusters



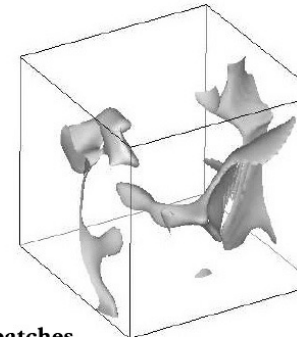
5 peak patches



10 peak patches



20 peak patches



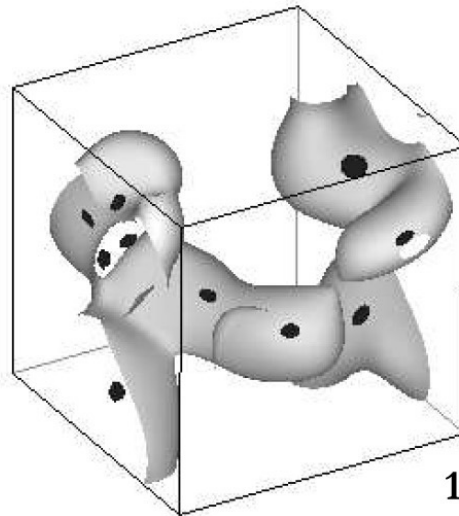
Tidal Shaping of the Cosmic Web

Cosmic Web Theory

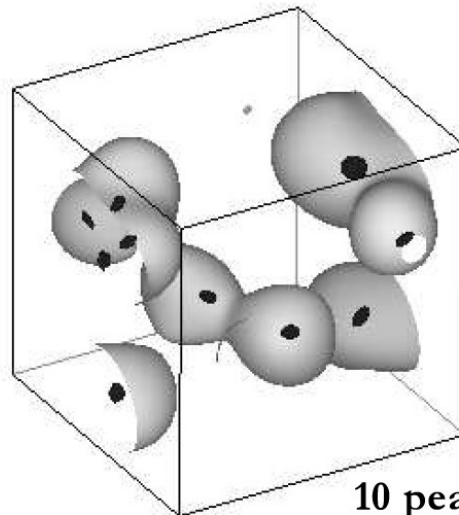
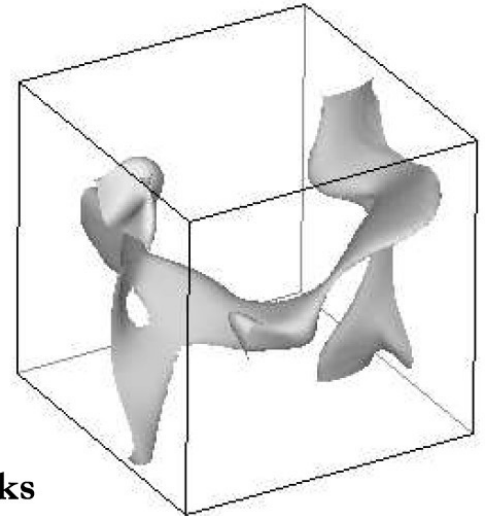
Bond, Kofman &
Pogosyan 1996

Strength of Filaments:

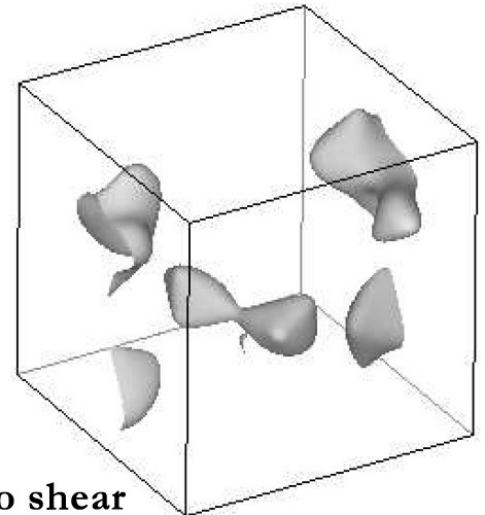
- Mass of clusters
- Distance between clusters
- Alignment between cluster shapes



10 peaks



10 peaks, no shear



Tidal Shaping of the Cosmic Web

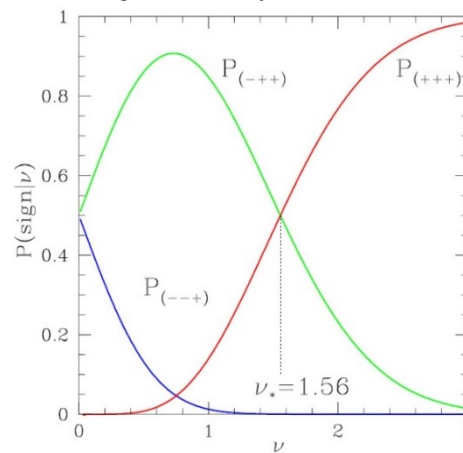
Cosmic Web Theory

Bond, Kofman &
Pogosyan 1996

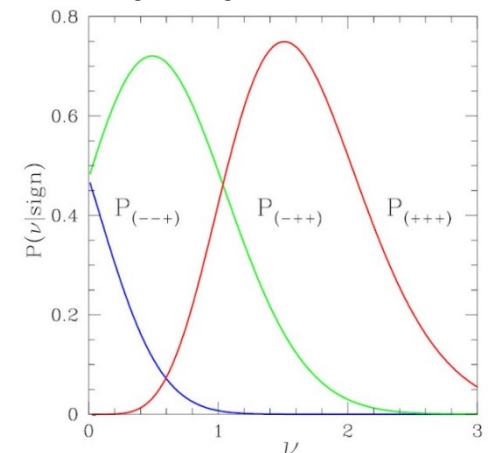
Deformation
Eigenvalue signature:

- --- voids
- --+ wall
- -++ filament
- +++ cluster node

Probability Deformation signature
for given density



Probability density
for given signature



Conditional Statistics

Tidal Shear eigenvalues in Gaussian field:

in overdense regions:

most prominent structures are FILAMENTS

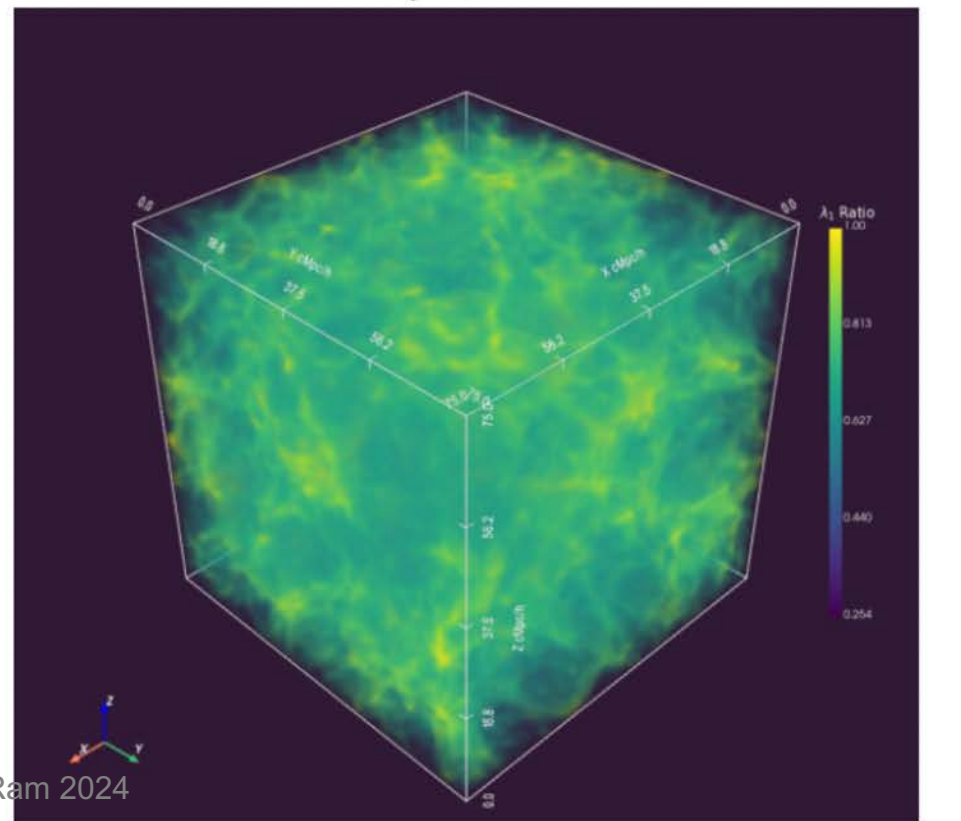
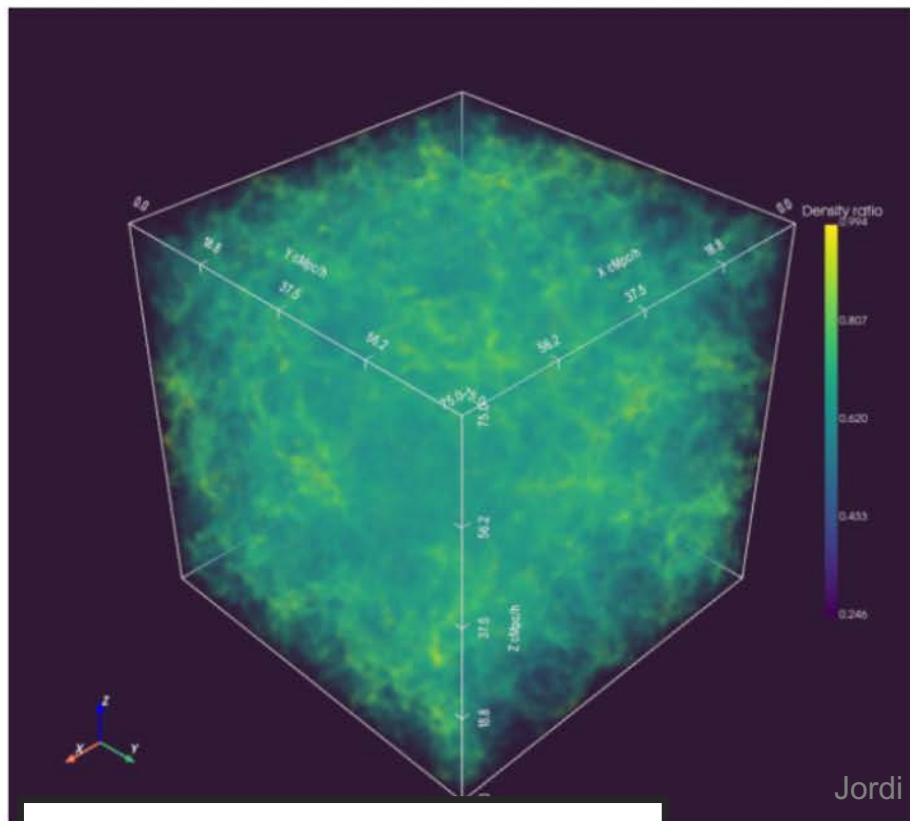
in underdense regions:

most prominent structure are WALLS

Tidal Shaping of the Cosmic Web

δ at $z = 6.0$

λ_1 at $z = 6.0$



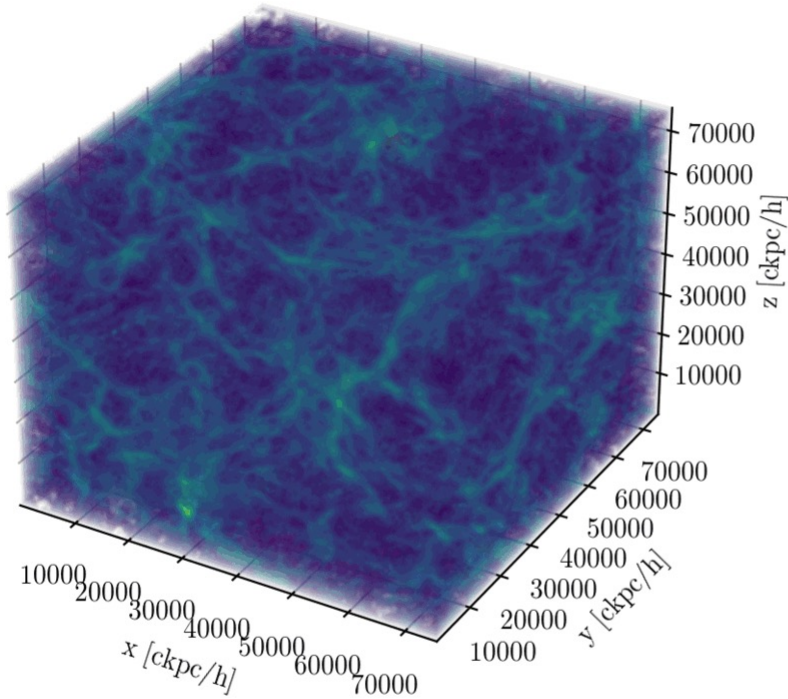
Jordi Ram 2024

Illustris-3-Dark
Z=6

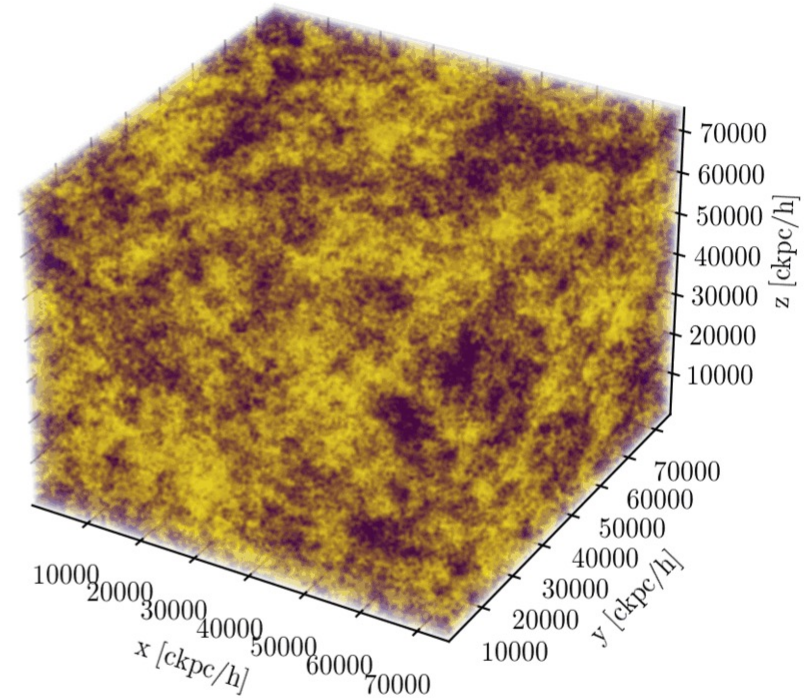
Alexandru Oprea 2025

Tidal Shaping of the Cosmic Web

First Eigenvalue Field at $z=46.77$

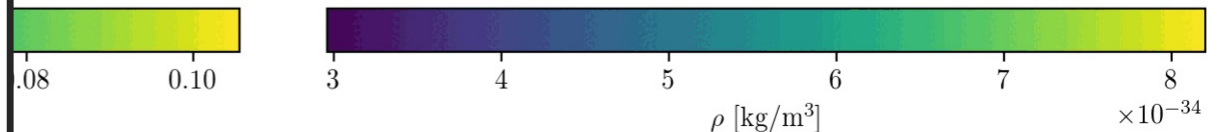


Density Field at $z=46.77$



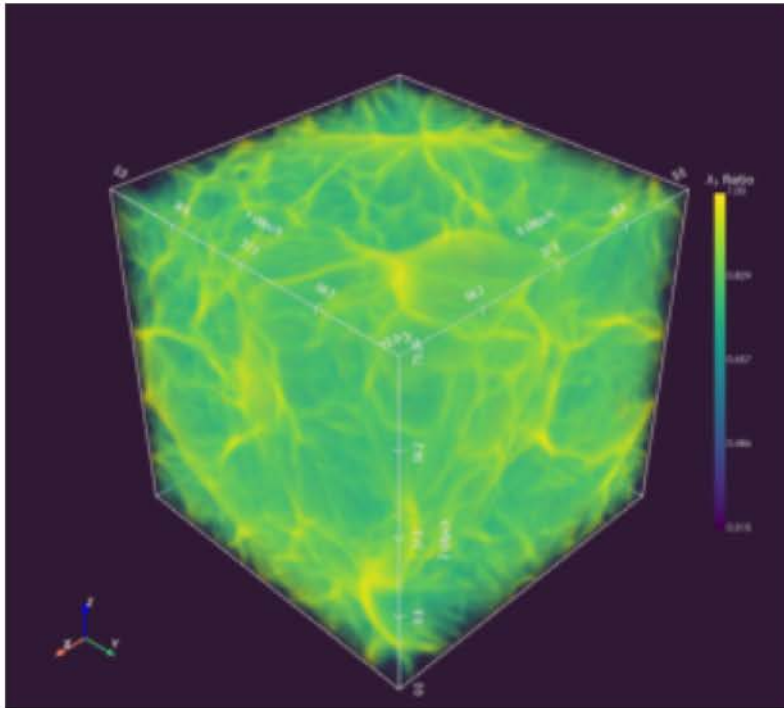
Jordi Ram 2024

**Tidal Forces
shape the Cosmic Web**

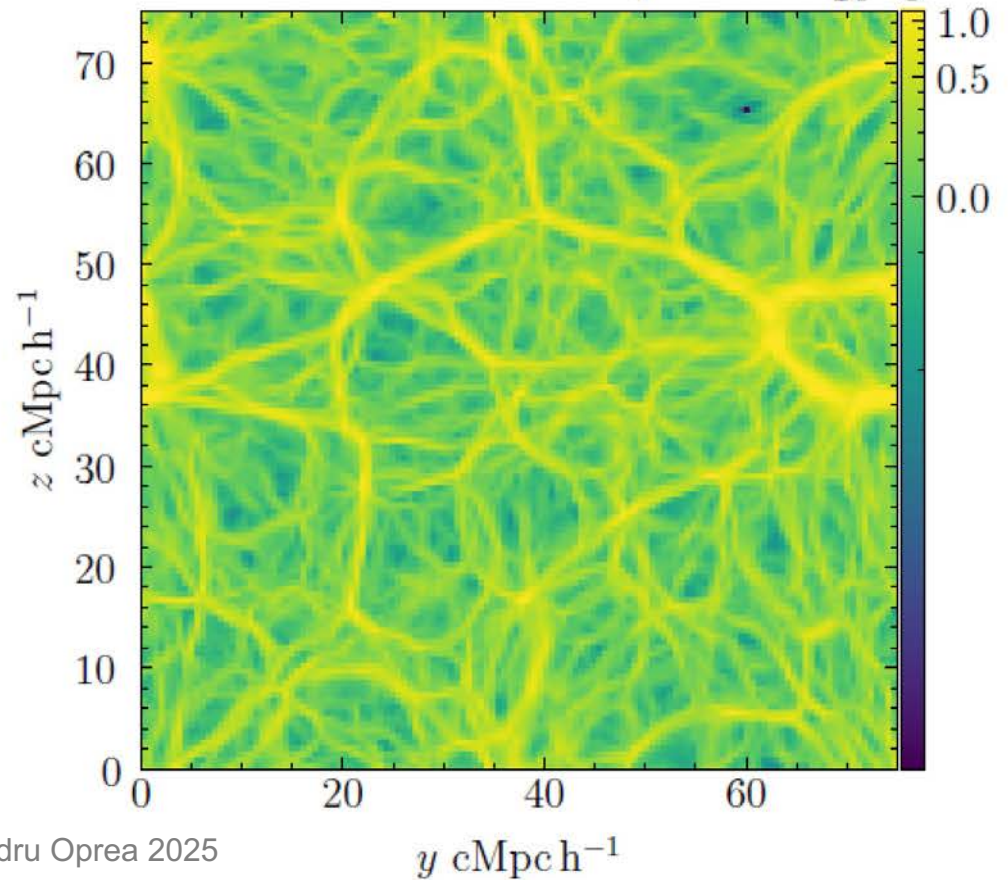


Tidal Shaping of the Cosmic Web

λ_1 (LOG) at $z = 0.0$



Slice at $x=36.9 \text{ cMpc h}^{-1}$ $1e-9$



Alexandru Oprea 2025

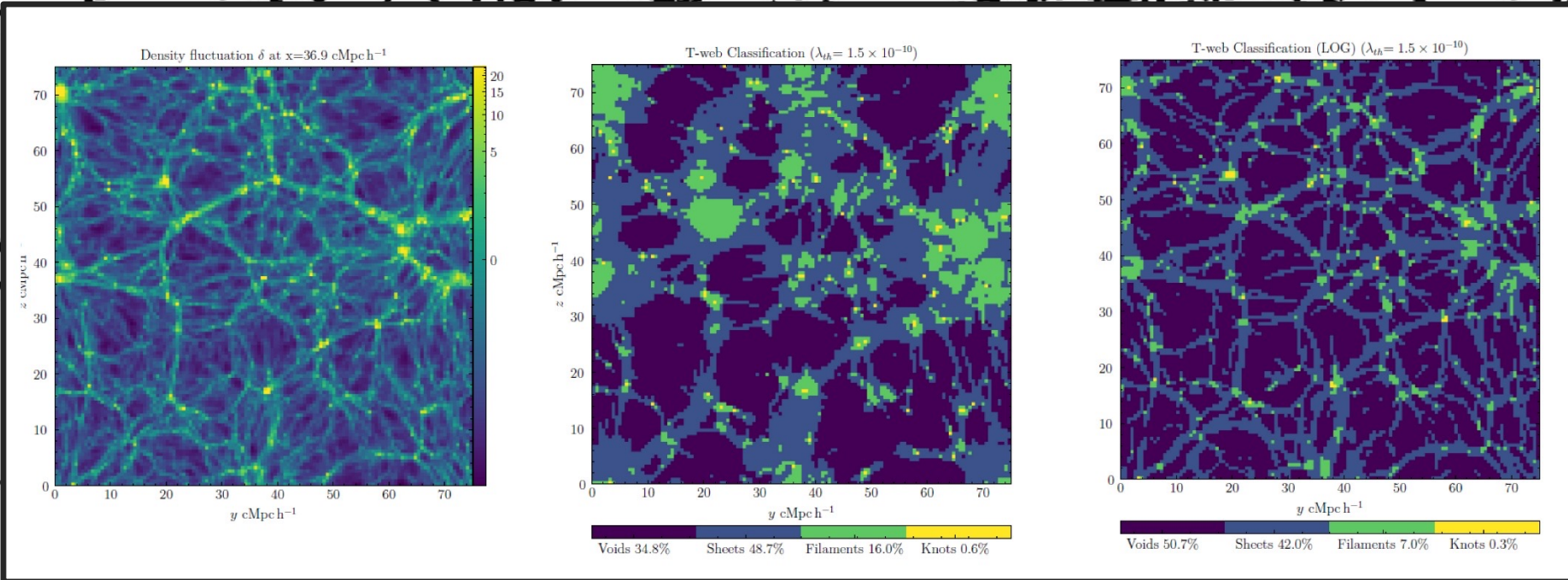
Poisson modified tidal field

$$\nabla^2 \Phi = 4\pi G \bar{\rho}_m a^2 \log_{10}(1 + \delta),$$

**Coherent weblike character
tidal force field**

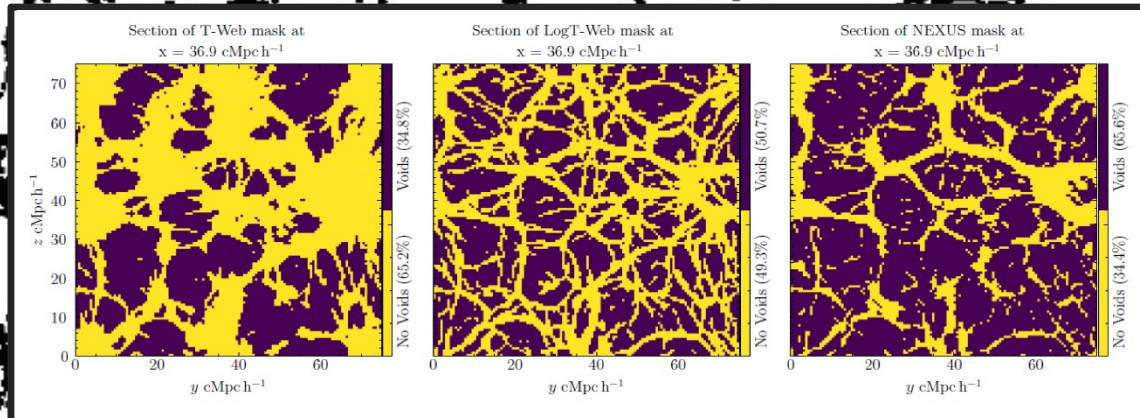


Tidal Classification of the Cosmic Web: Tweb



Tidal Eigenvalue signature:

- voids
- + wall
- ++ filament
- +++ cluster node

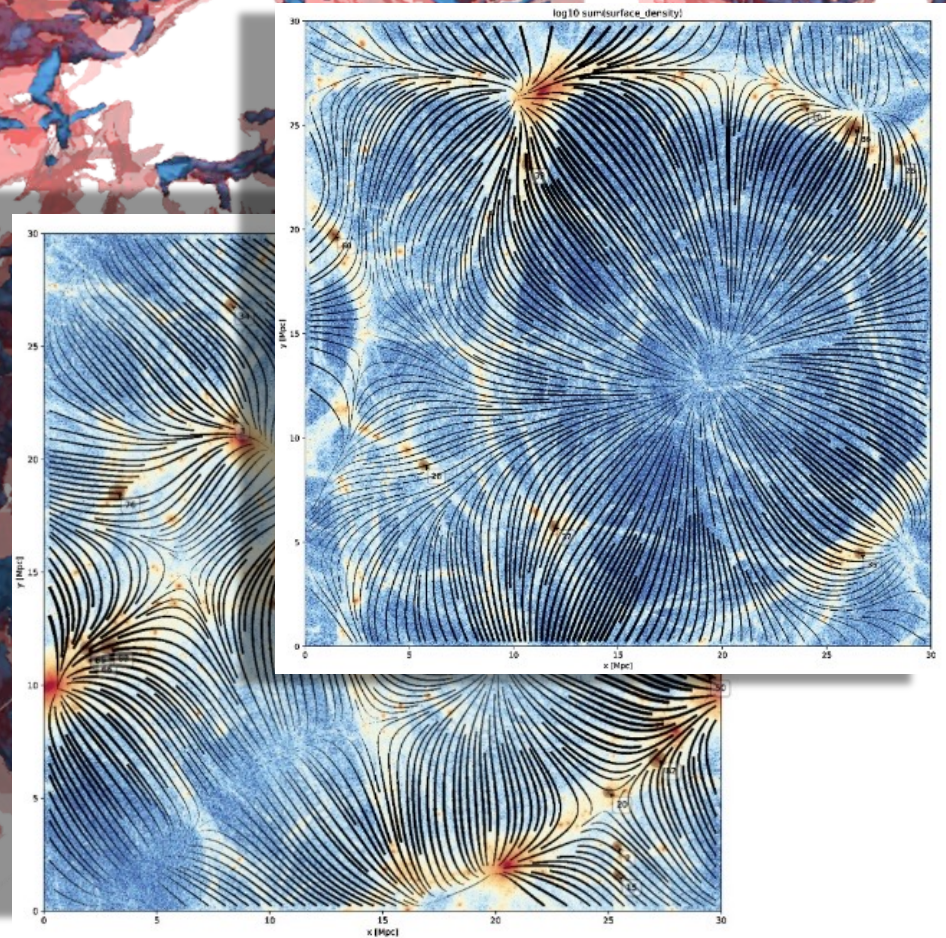
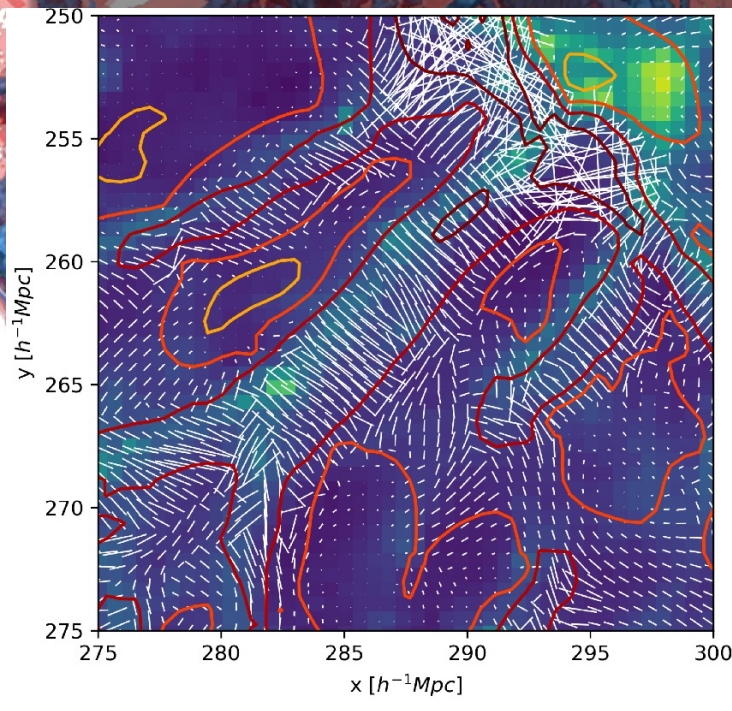


Cosmic Web

Force Field Inventory

Cosmic Web

Forces & Strains



Cosmic Web: Force & Flow Definition

Force and Velocity

$$\vec{g}(\vec{x}) = \frac{3\Omega_m H_0^2}{8\pi} \int d\vec{x}' \Delta_m(\vec{x}') \frac{\vec{x}' - \vec{x}}{|\vec{x}' - \vec{x}|^3}$$

Discretize

$$\vec{g}_{i,j}(\vec{r}) = \frac{3\Omega_m H_0^2}{8\pi} \sum_i^N \frac{\Delta_{m,i}(\vec{r} - \vec{r}_i')}{(\vec{r} - \vec{r}_i')^3} \hat{\delta}_j^{NEXUS}$$

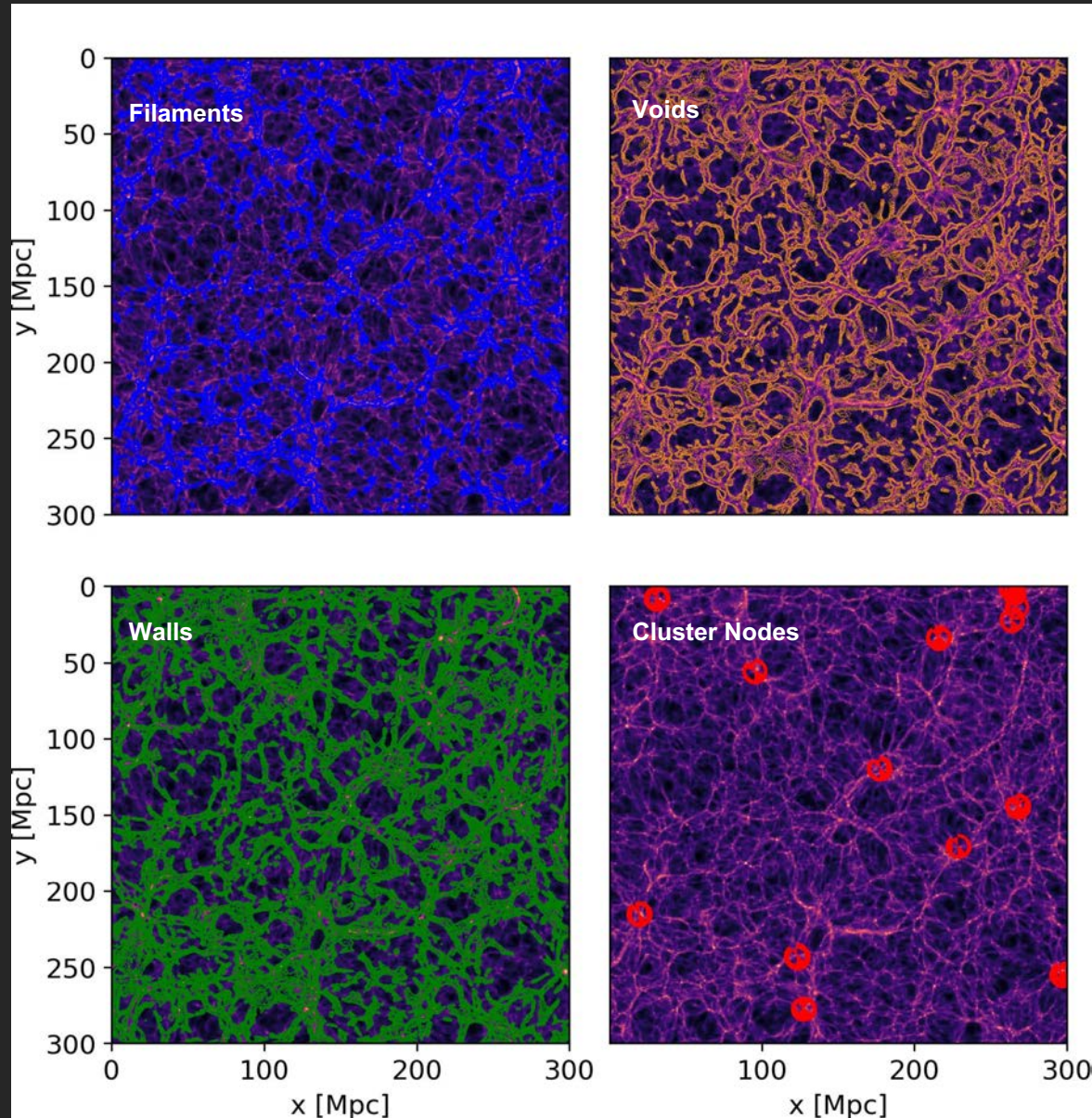
Linearize
Peebles (1980)

$$\vec{v}_{lin} = \frac{2f}{3H_0\Omega_m} \vec{g}$$

NEXUS+

Aragon-Calvo et al. 2007, Cautun et al. 2013, 2014

Nexus+ components



Cosmic Web: Force & Flow Definition

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Linearize

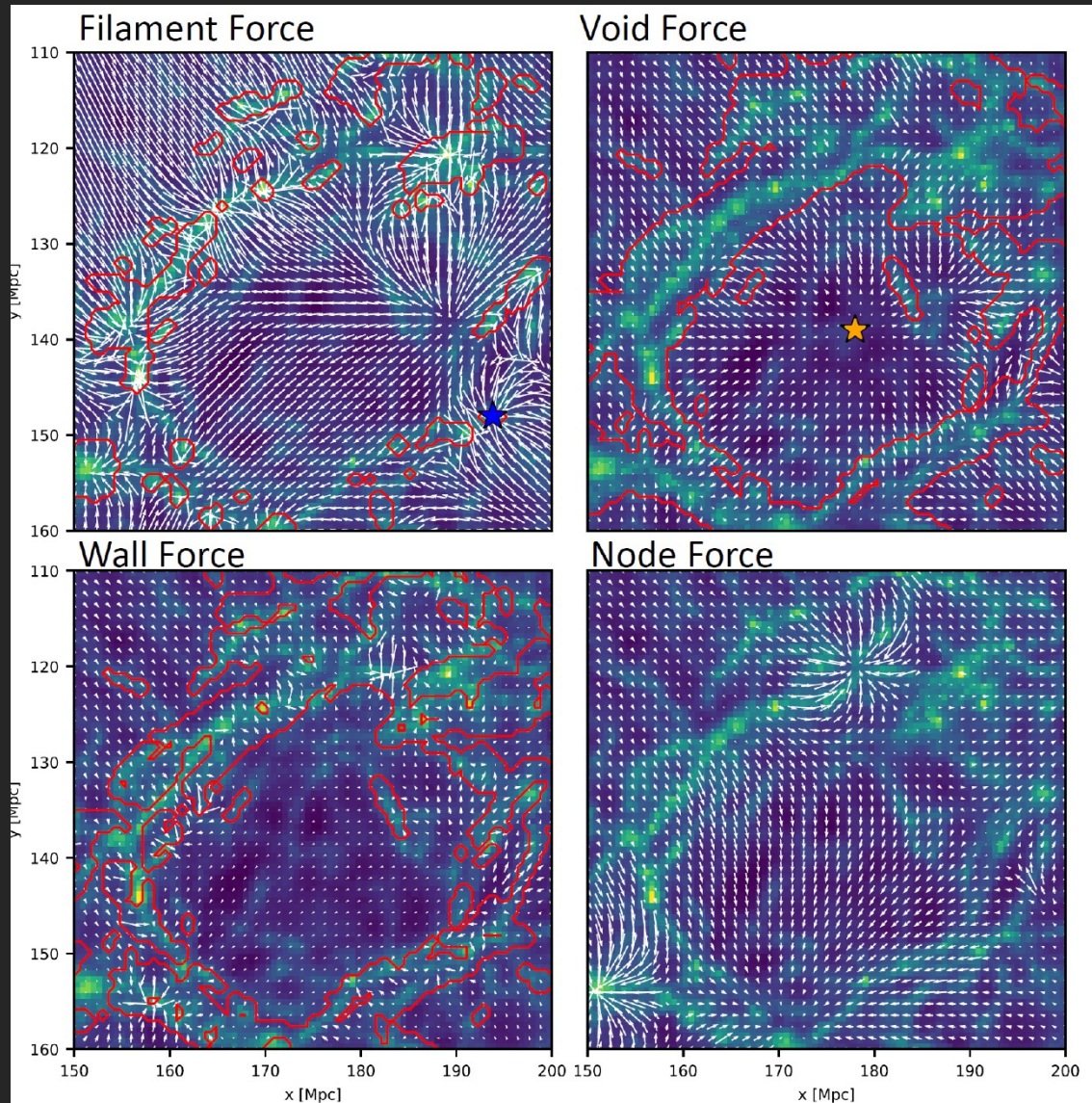
Peebles (1980)

$$\vec{v}_{lin} = \frac{2f}{3H_0\Omega_m} \vec{g}$$

NEXUS+

Aragon-Calvo et al. 2007, Cautun et al. 2013, 2014

Force field - vector



Cosmic Web: Force & Flow Definition

Force and Velocity

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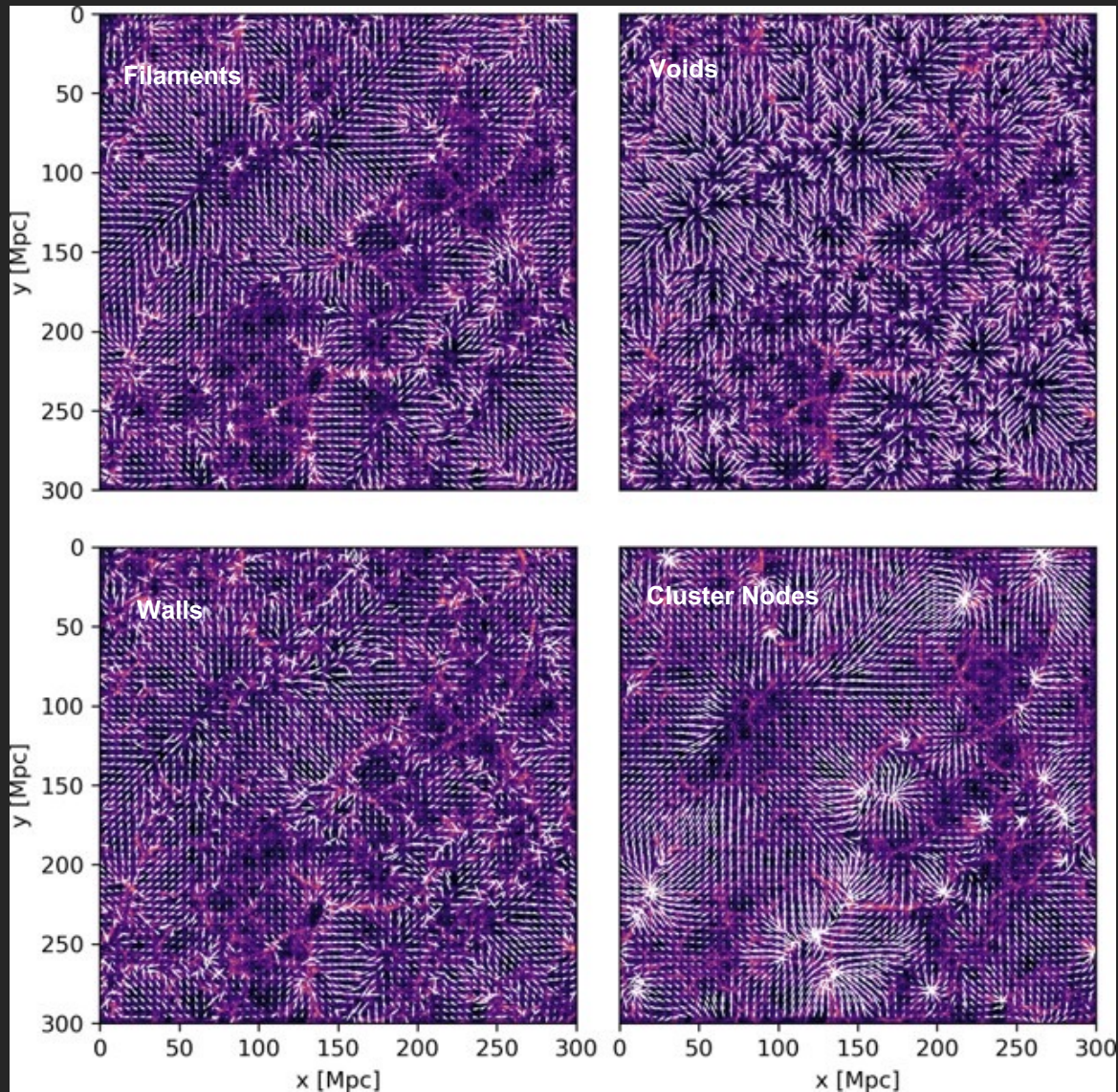
Linearize
Peebles (1980)

$$\vec{v}_{lin} = \frac{2f}{3H_0\Omega_m} \vec{g}$$

NEXUS+

Aragon-Calvo et al. 2007, Cautun et al. 2013, 2014

Force field - vector



Cosmic Web: Force & Flow Definition

Force and Velocity

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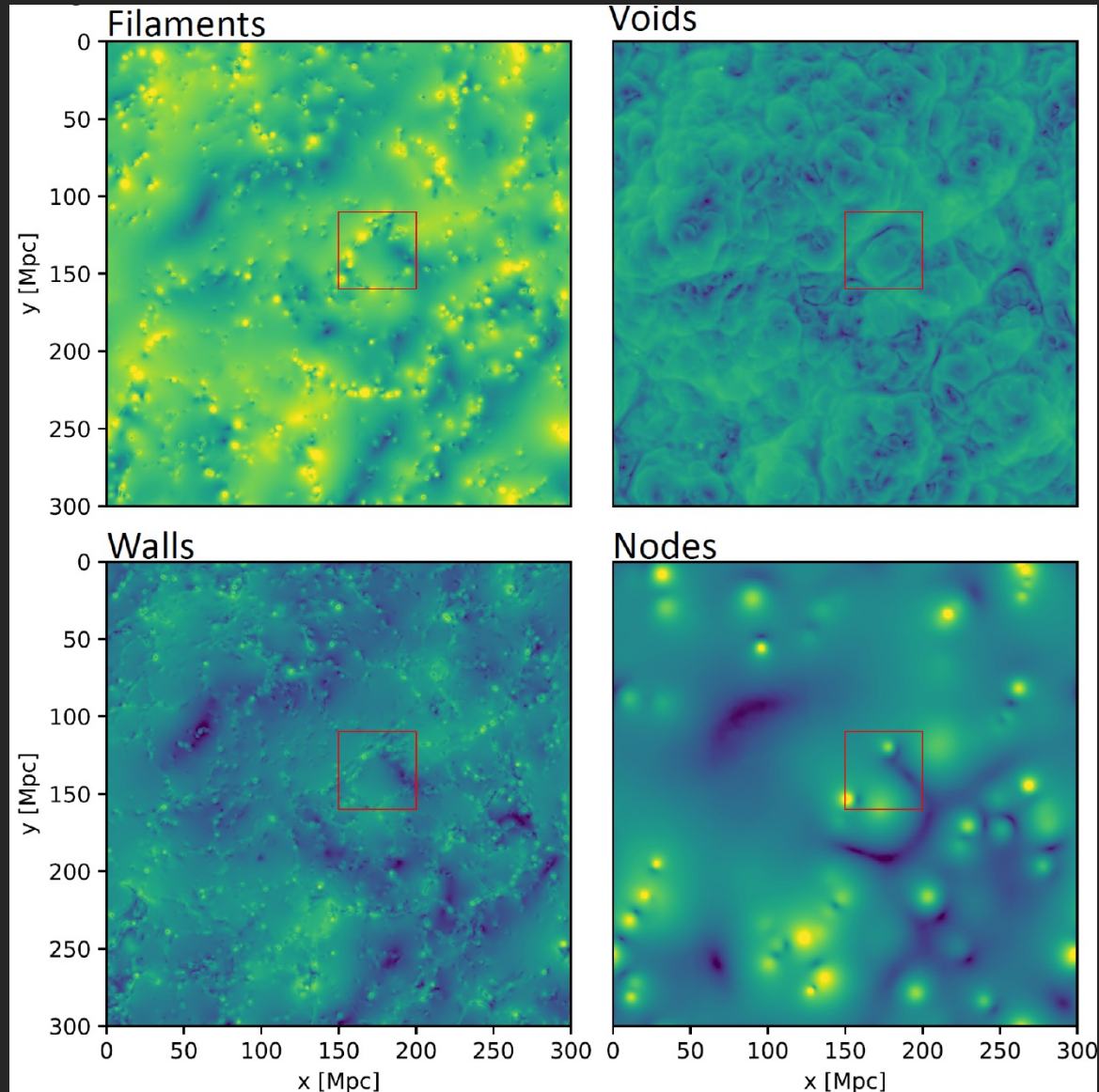
Linearize
Peebles (1980)

$$\vec{v}_{lin} = \frac{2f}{3H_0\Omega_m} \vec{g}$$

NEXUS+

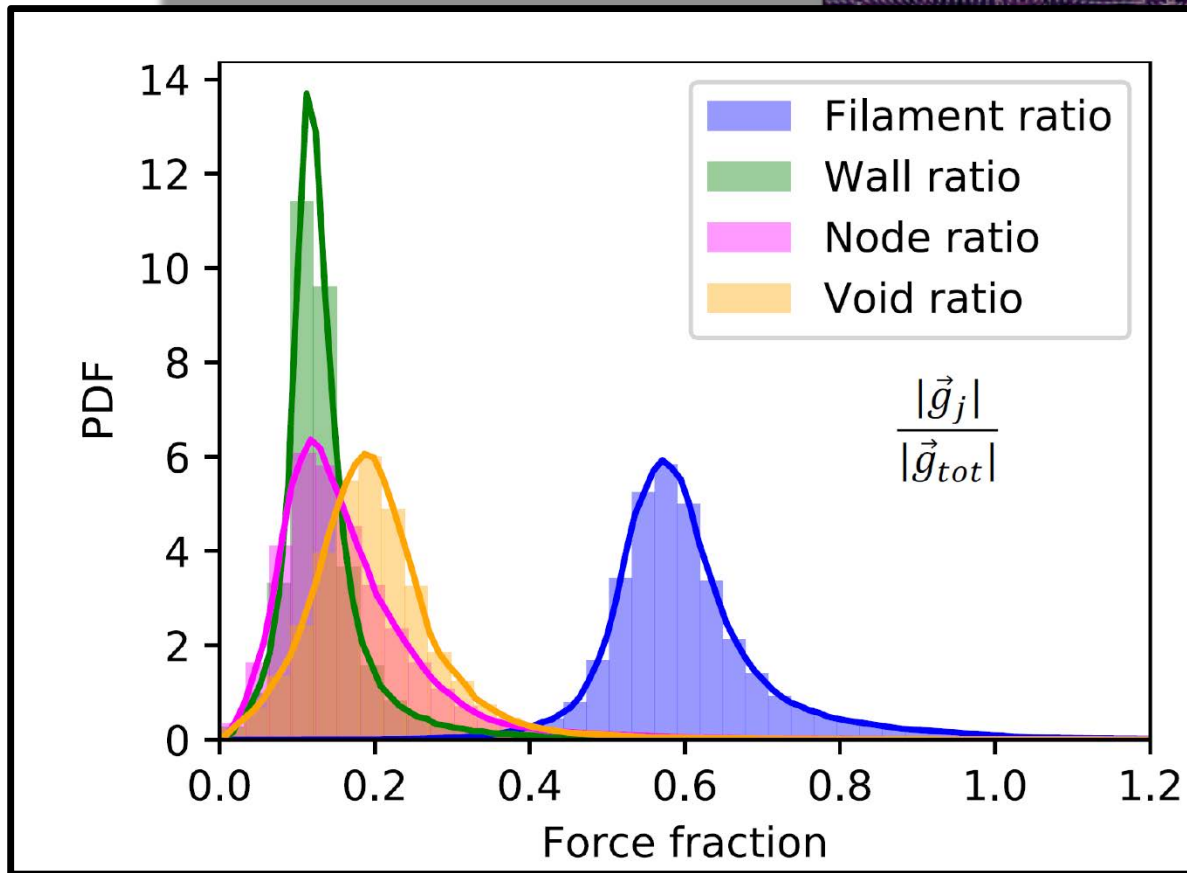
Aragon-Calvo et al. 2007, Cautun et al. 2013, 2014

Force field - amplitude



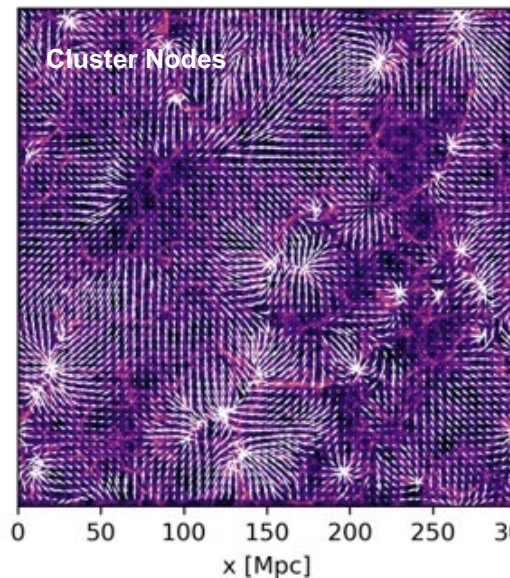
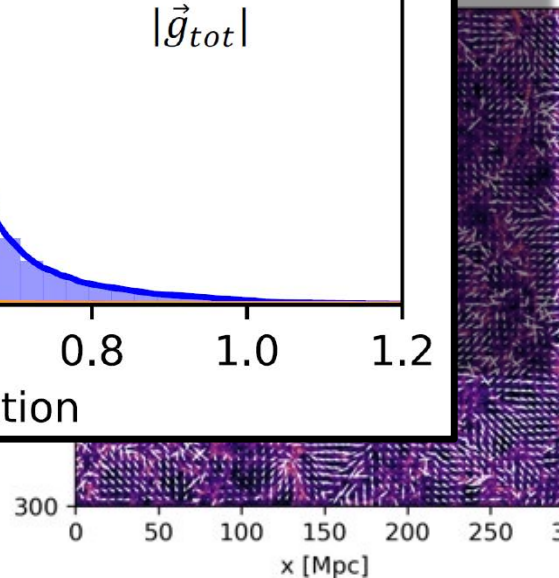
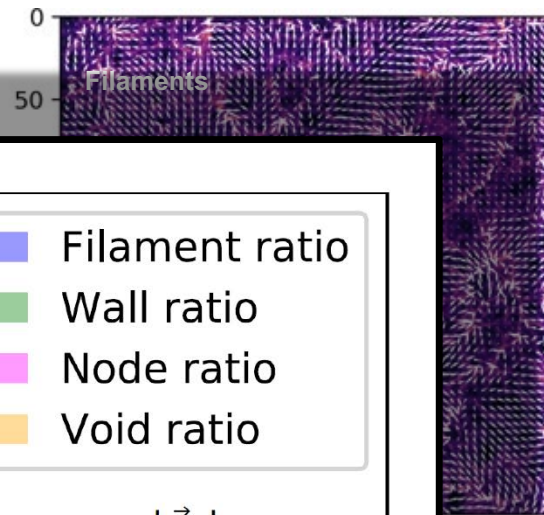
Force Field Inventory

Filaments dominate force field

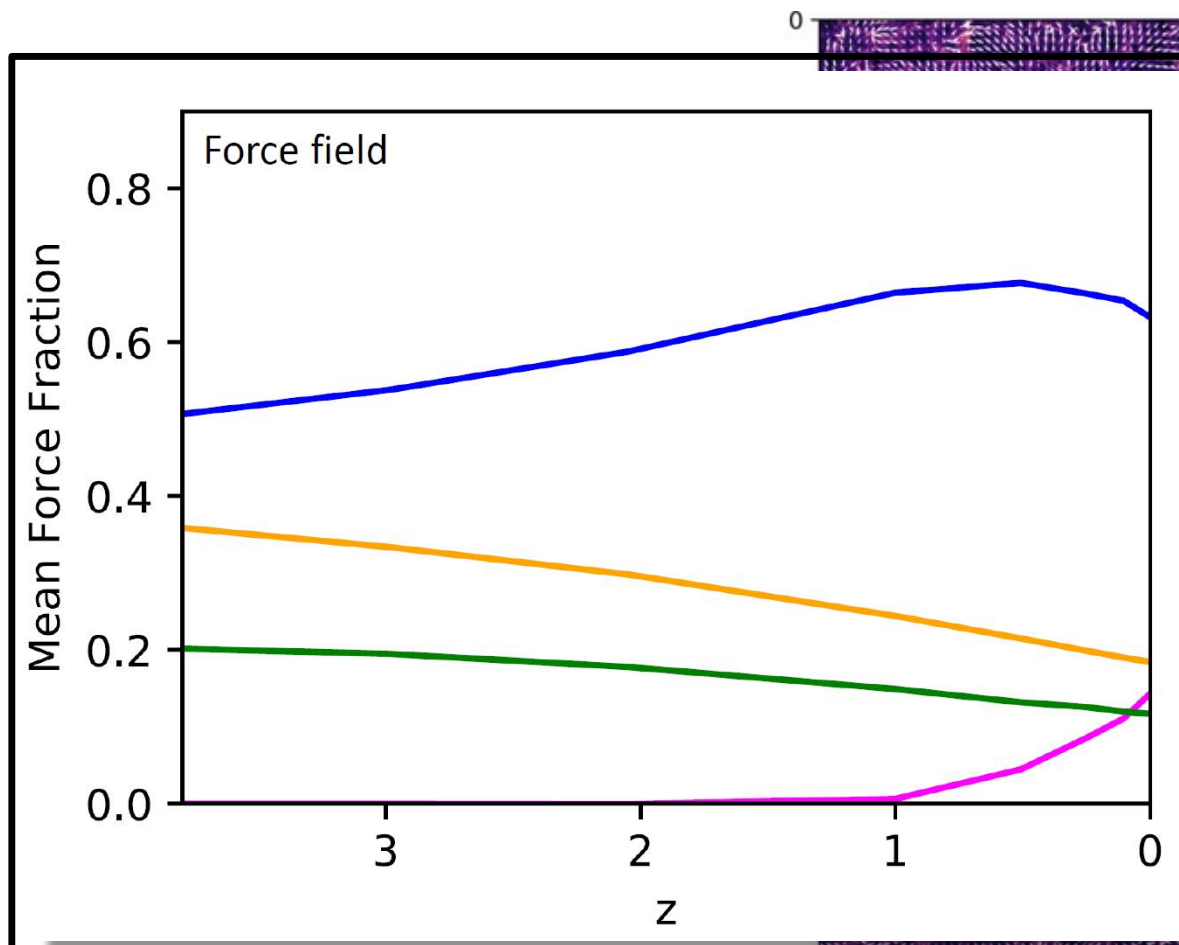


Force ranking:

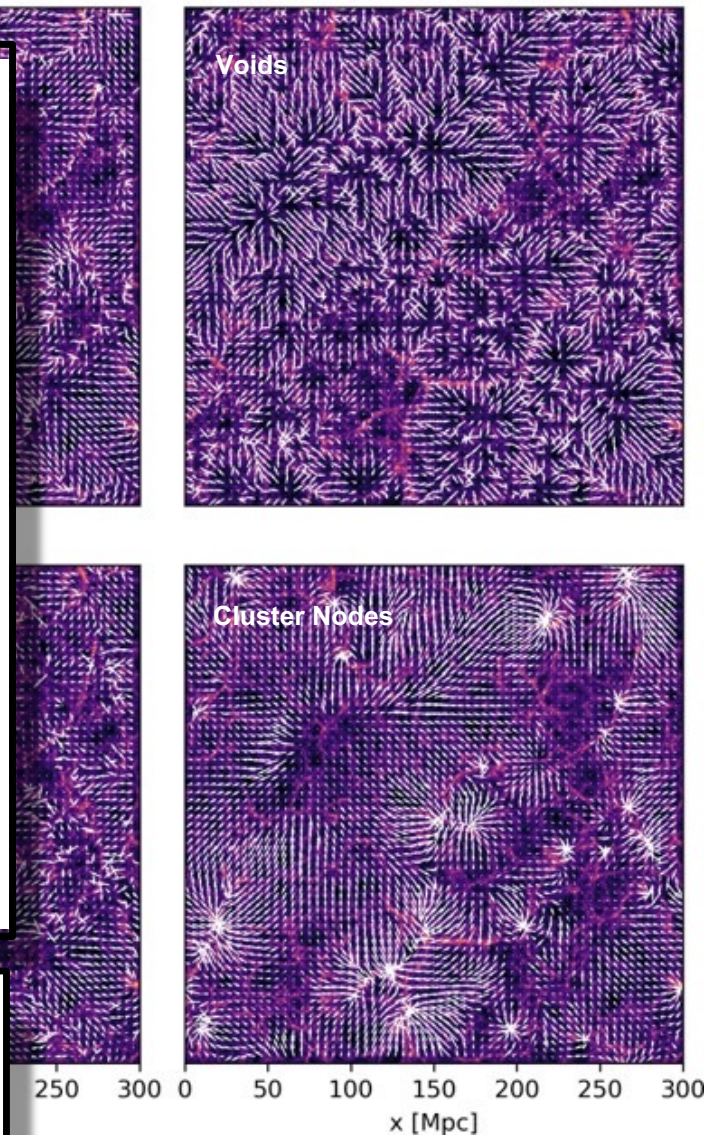
Filaments – Voids –
Cluster Nodes - Walls



Force Field Evolution



- Filaments dominant throughout cosmic evolution
- Voids much more influential at high z
- Clusters recently quickly grow more influential



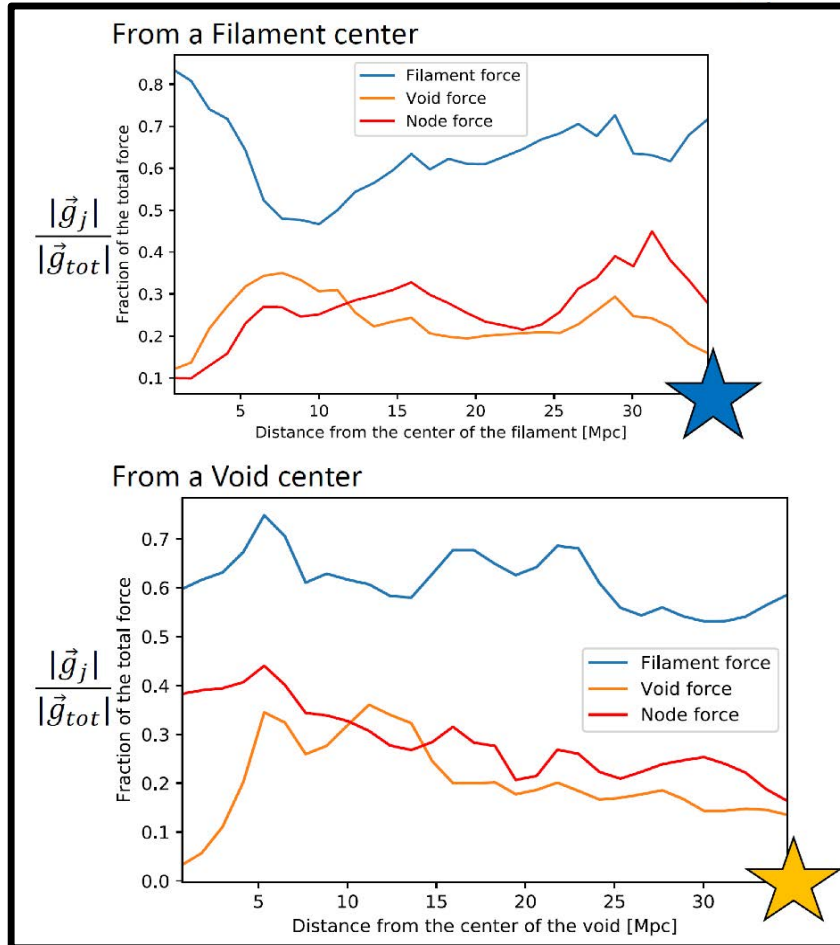
Cosmic Web:

Flow & Force Field

Case Studies

Filament vs. Void Forces

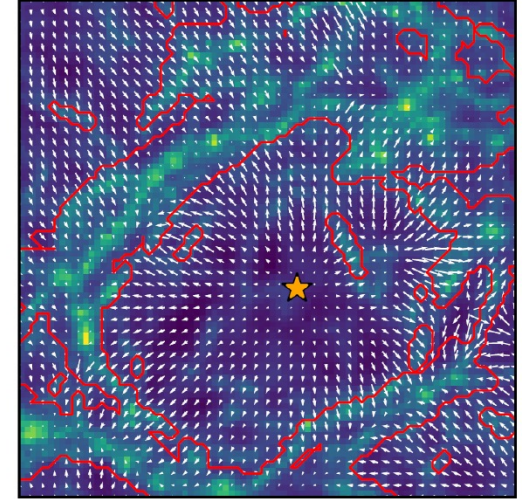
Filament vs. Void profile



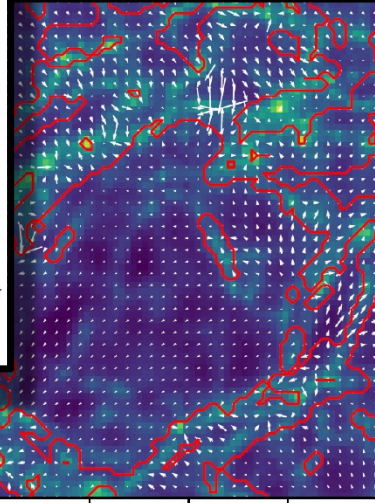
Filament Force



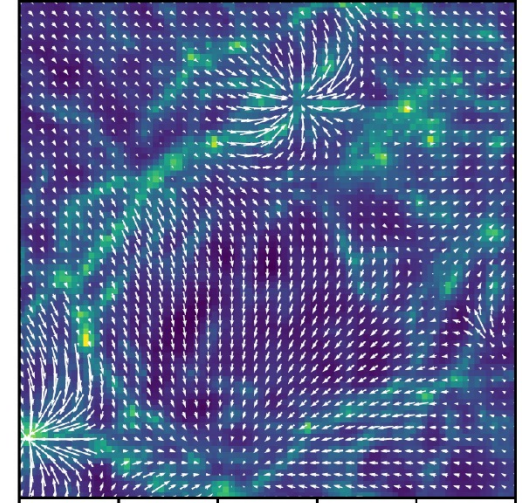
Void Force



Force



Node Force



Case study:

- Filaments dominate nearly everywhere
- Voids strong in interior
- Clusters may dominate over voids

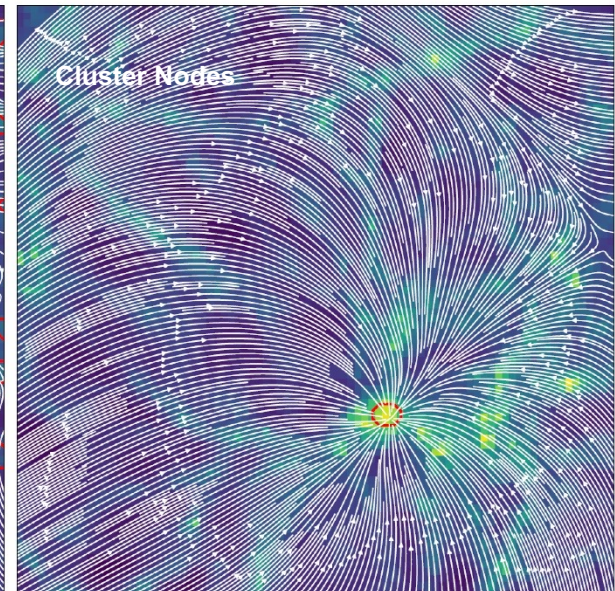
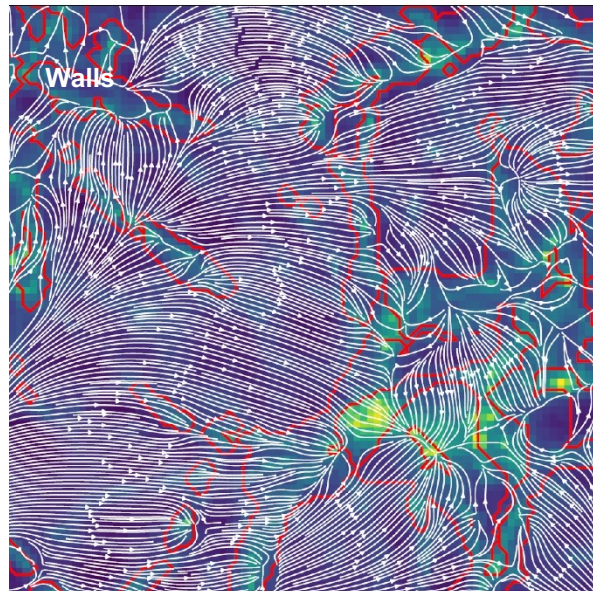
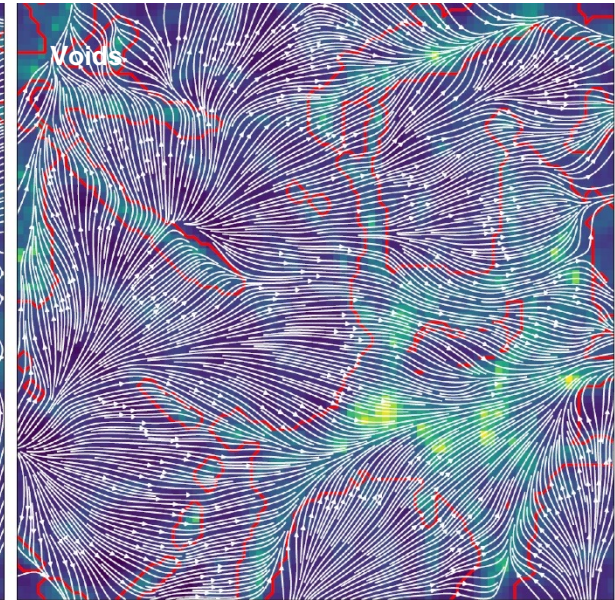
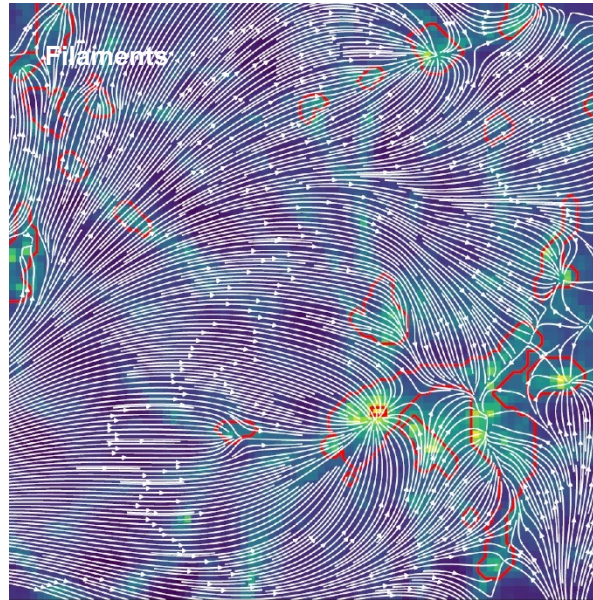
Flows around Cluster Nodes

Cluster Node environment:

Flowlines:

- **velocity field contribution**

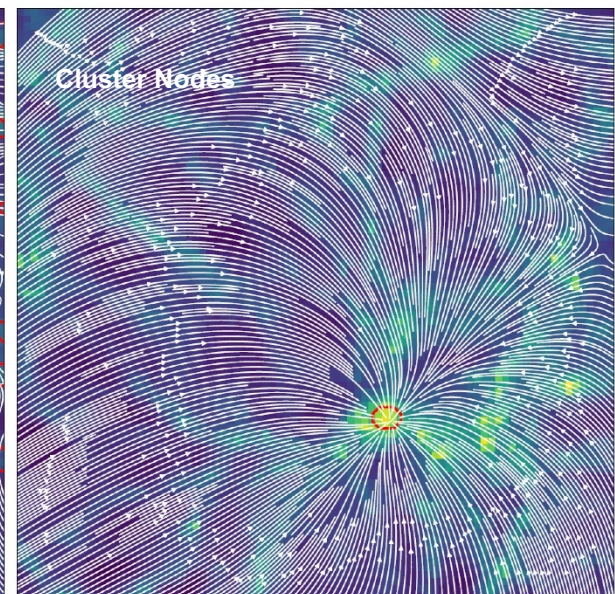
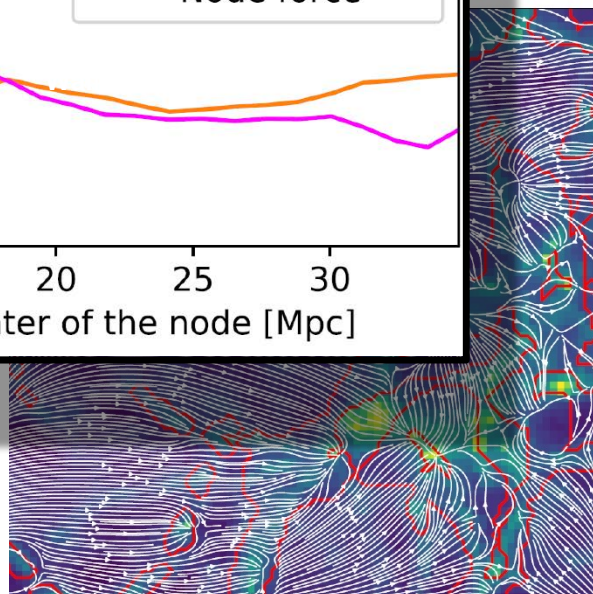
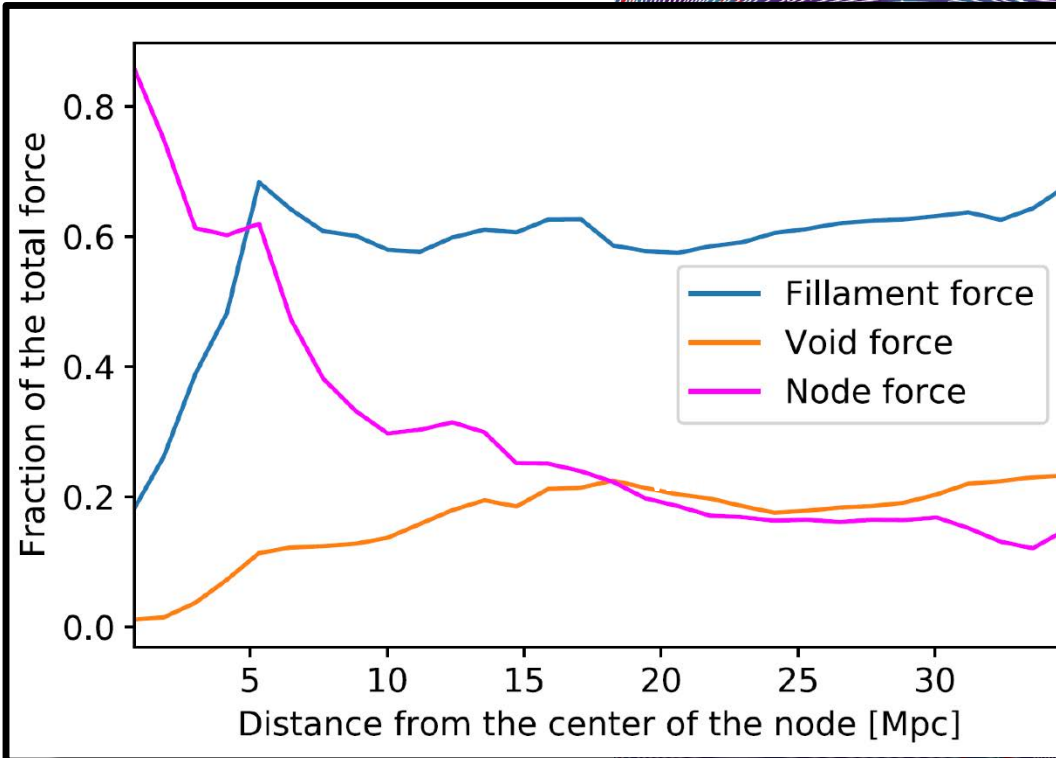
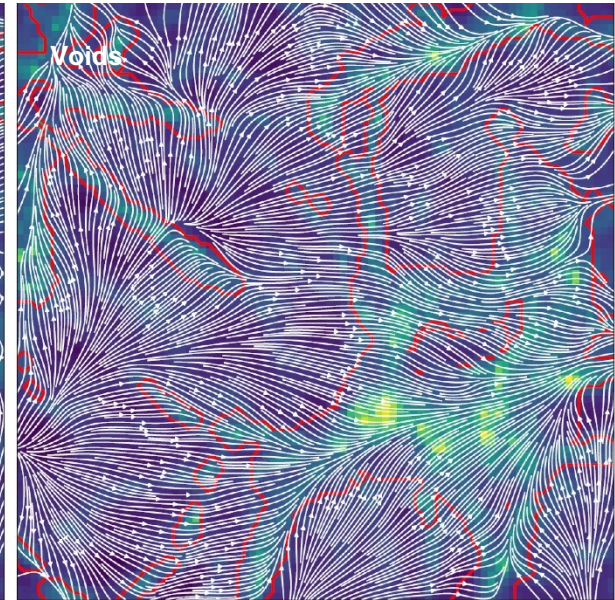
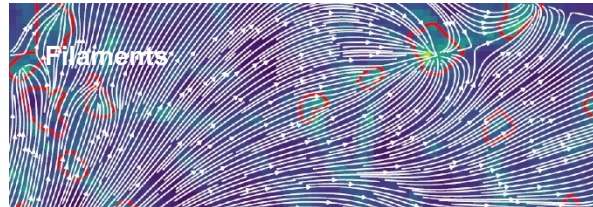
**individual
cosmic web components**



Flows around Cluster Nodes

Force Field around Cluster Node:

- Cluster dominant: $r < 5$ Mpc
- Void over Cluster: $r > 15$ Mpc

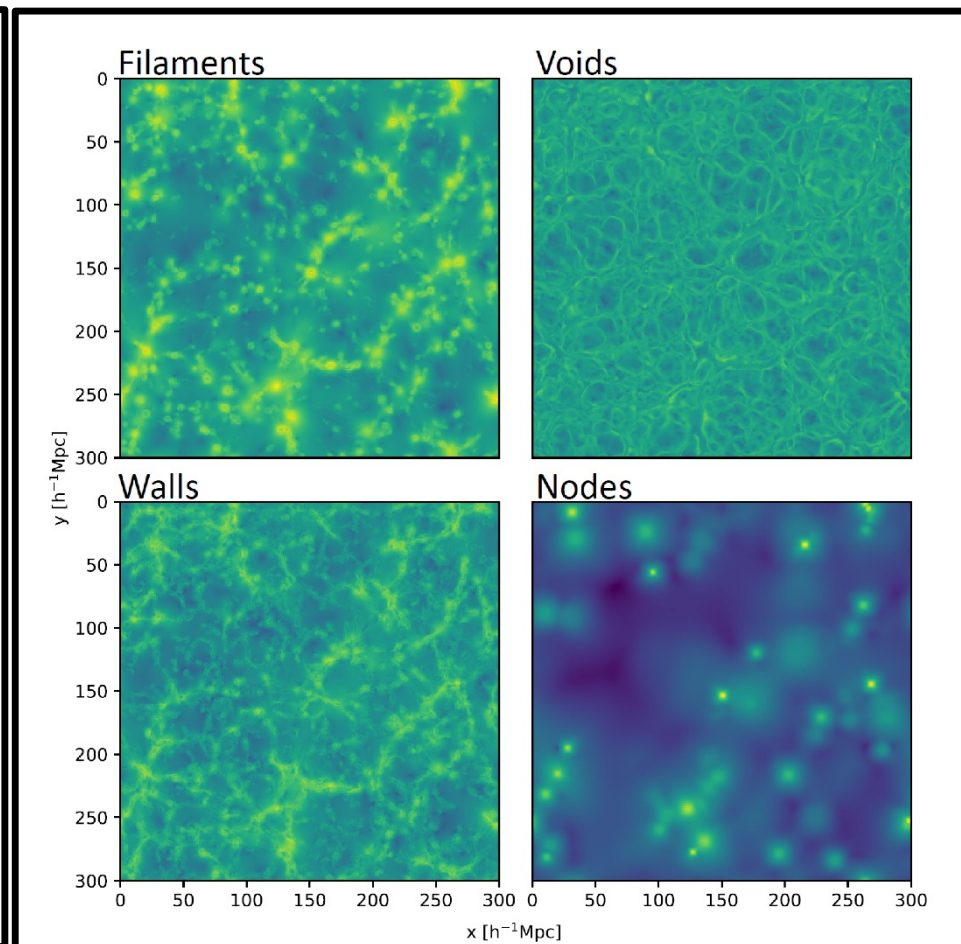
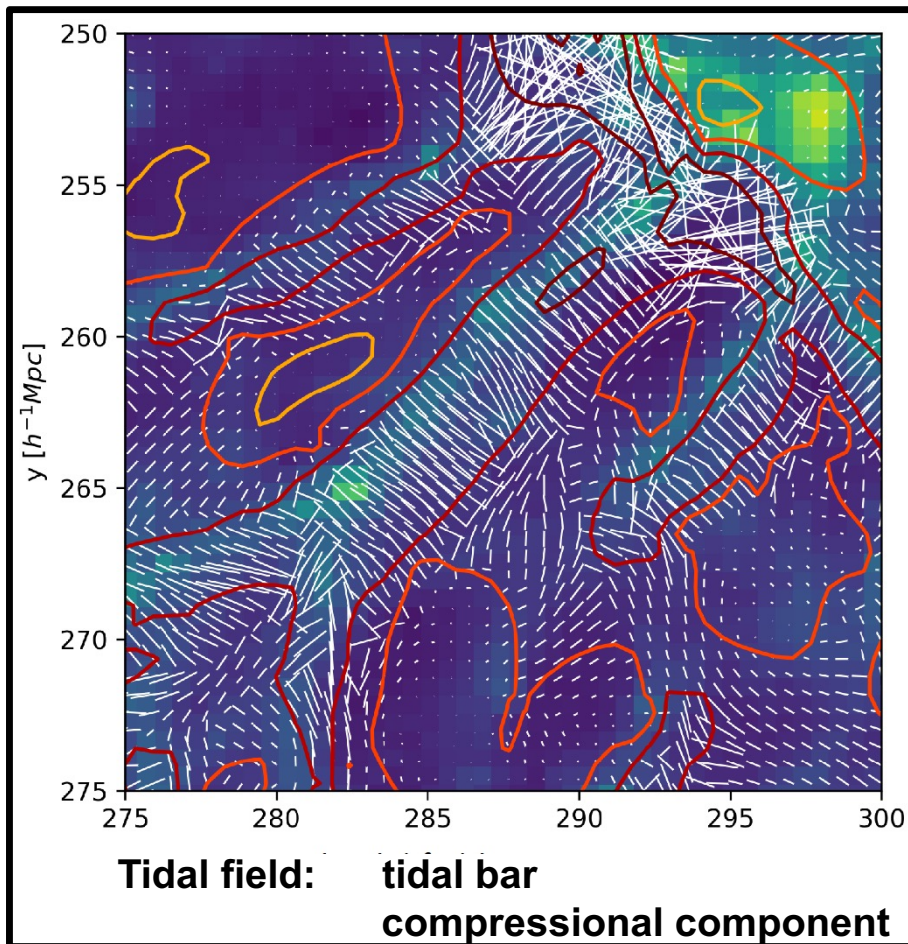


Cosmic Web

Tidal Field Inventory

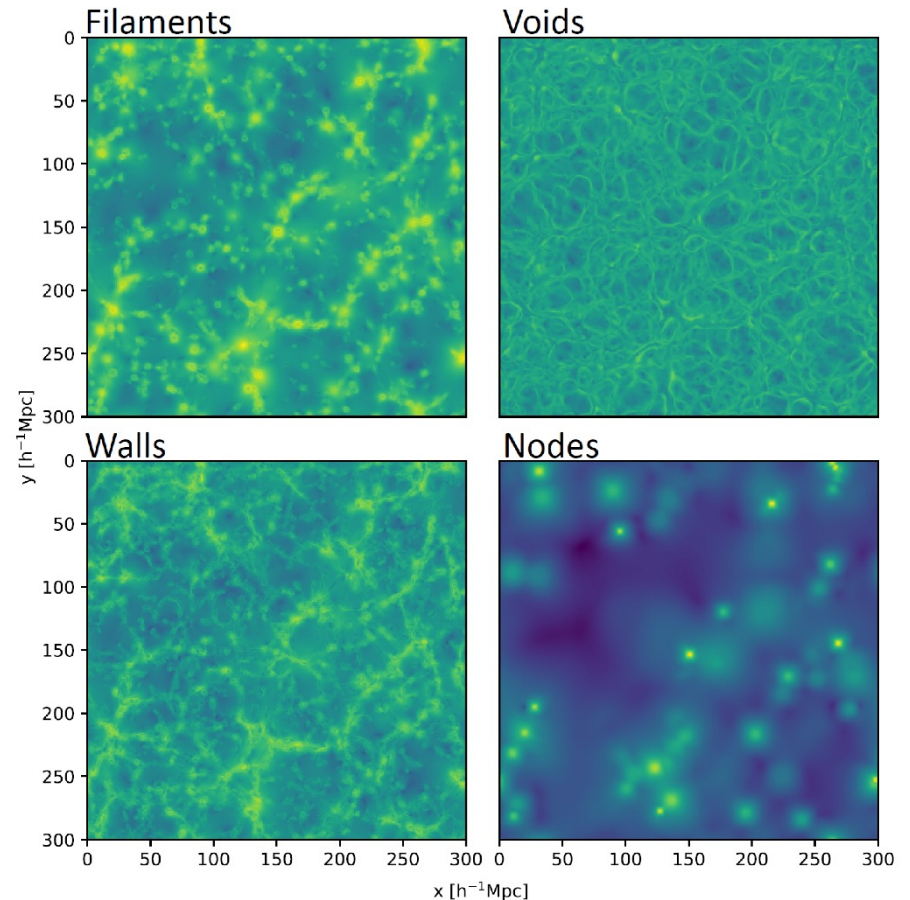
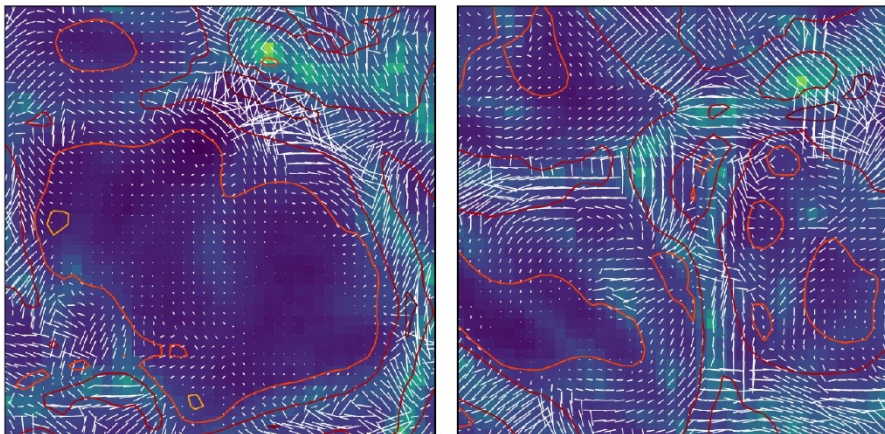
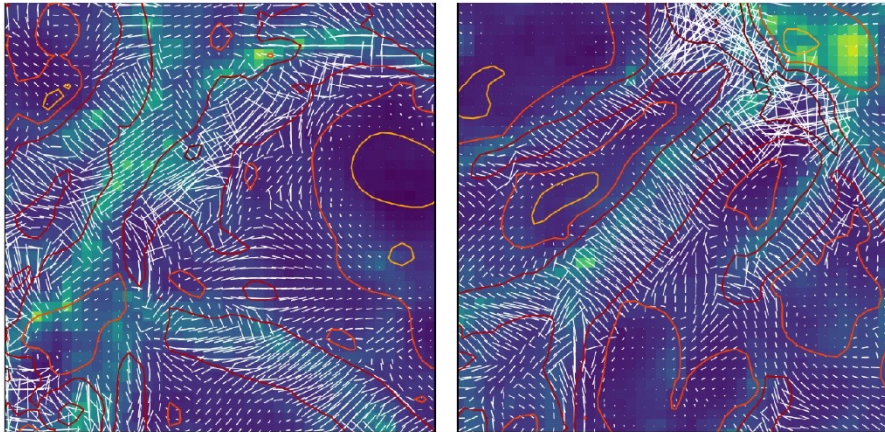
Tidal Field: Definition

$$T_{ij}(\vec{r}) = \frac{3\Omega_m H_0^2}{8\pi} \int d\vec{r}' \delta_m(\vec{r}' - \vec{r}) \frac{3(\vec{r}'_i - \vec{r}_i)(\vec{r}'_j - \vec{r}_j) - |\vec{r}' - \vec{r}|^2 \delta_{ij}}{|\vec{r}' - \vec{r}|^5} - \frac{1}{2} \Omega H^2 \delta(\vec{r}) \delta_{ij}$$



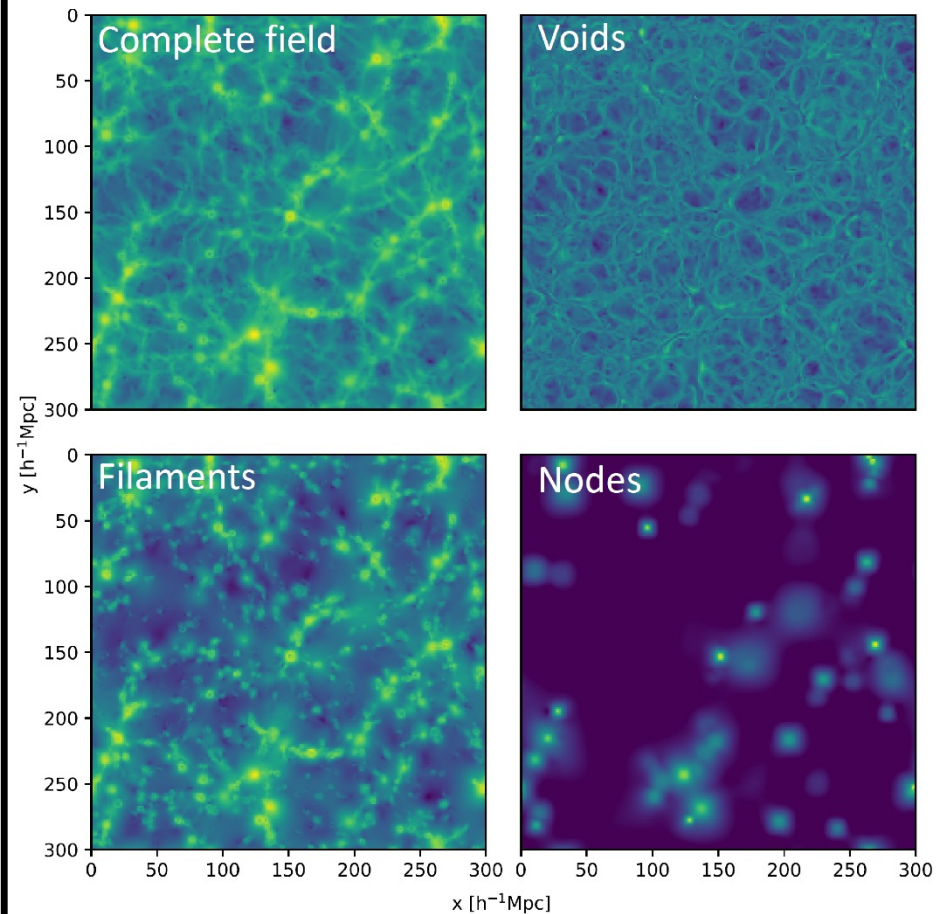
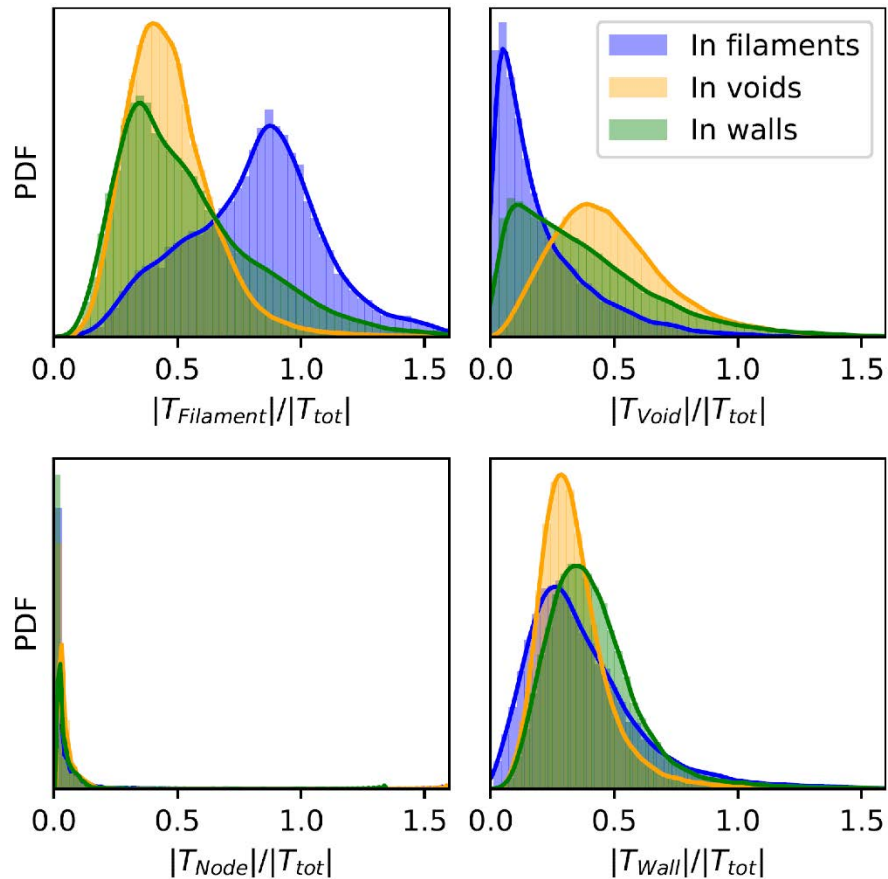
Tidal Field: Definition

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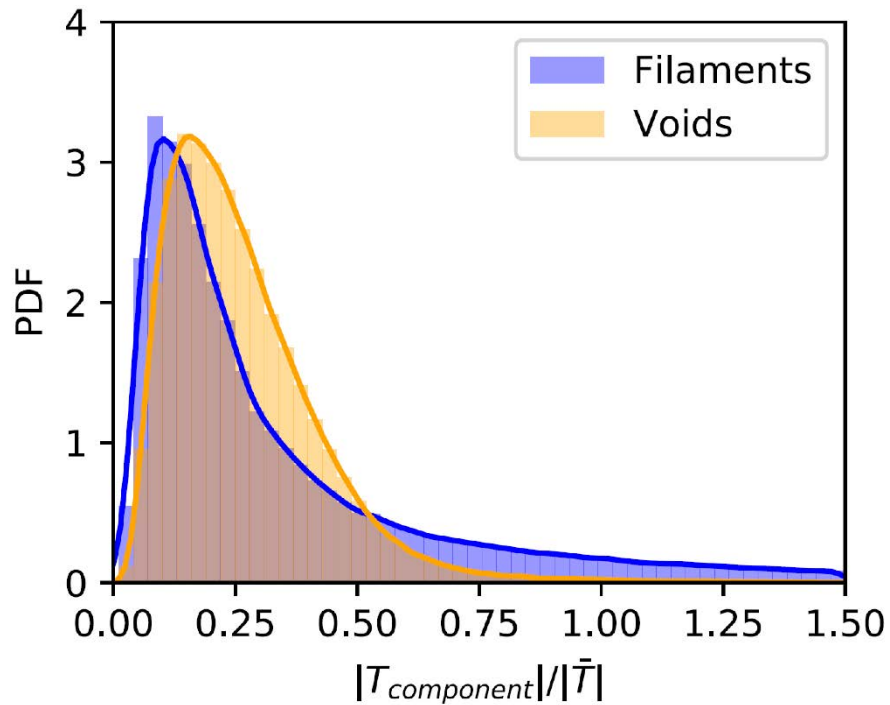
Tidal Field: Inventory

$$T_{ij}(\vec{r}) = \frac{3\Omega_m H_0^2}{8\pi} \int d\vec{r}' \delta_m(\vec{r}' - \vec{r}) \frac{3(\vec{r}'_i - \vec{r}_i)(\vec{r}'_j - \vec{r}_j) - |\vec{r}' - \vec{r}|^2 \delta_{ij}}{|\vec{r}' - \vec{r}|^5} - \frac{1}{2} \Omega H^2 \delta(\vec{r}) \delta_{ij}$$

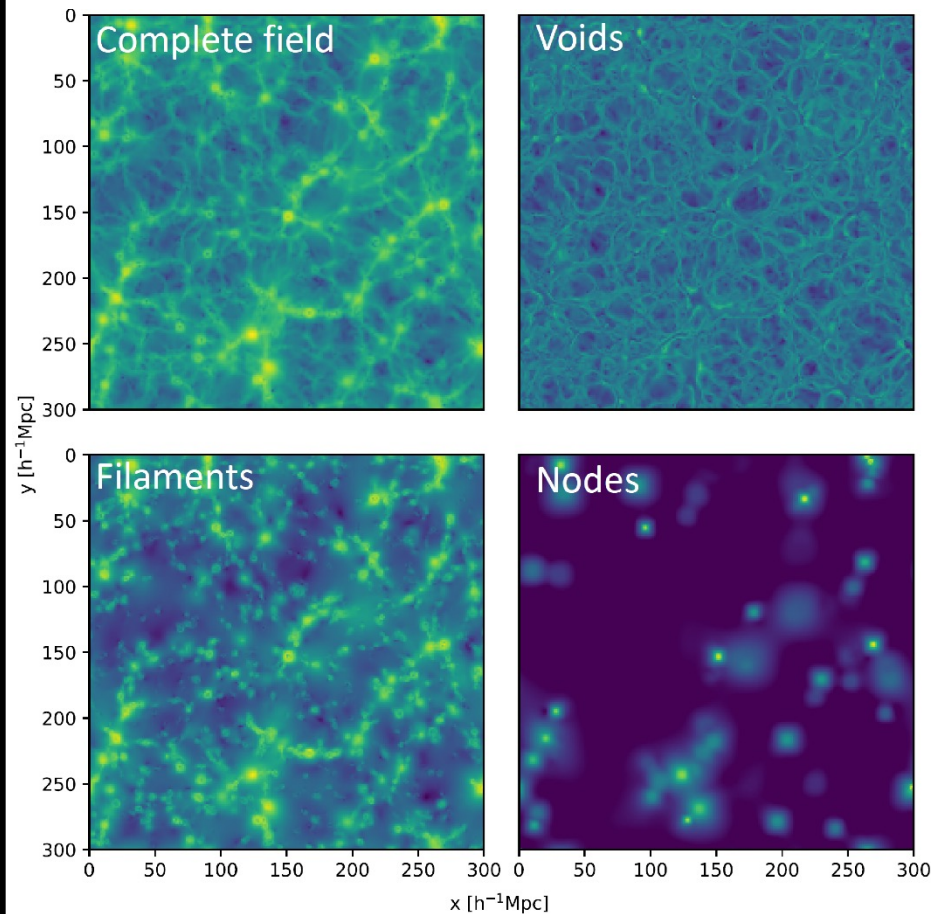


Tidal Field: Inventory

$$T_{ij}(\vec{r}) = \frac{3\Omega_m H_0^2}{8\pi} \int d\vec{r}' \delta_m(\vec{r}' - \vec{r}) \frac{3(\vec{r}'_i - \vec{r}_i)(\vec{r}'_j - \vec{r}_j) - |\vec{r}' - \vec{r}|^2 \delta_{ij}}{|\vec{r}' - \vec{r}|^5} - \frac{1}{2} \Omega H^2 \delta(\vec{r}) \delta_{ij}$$

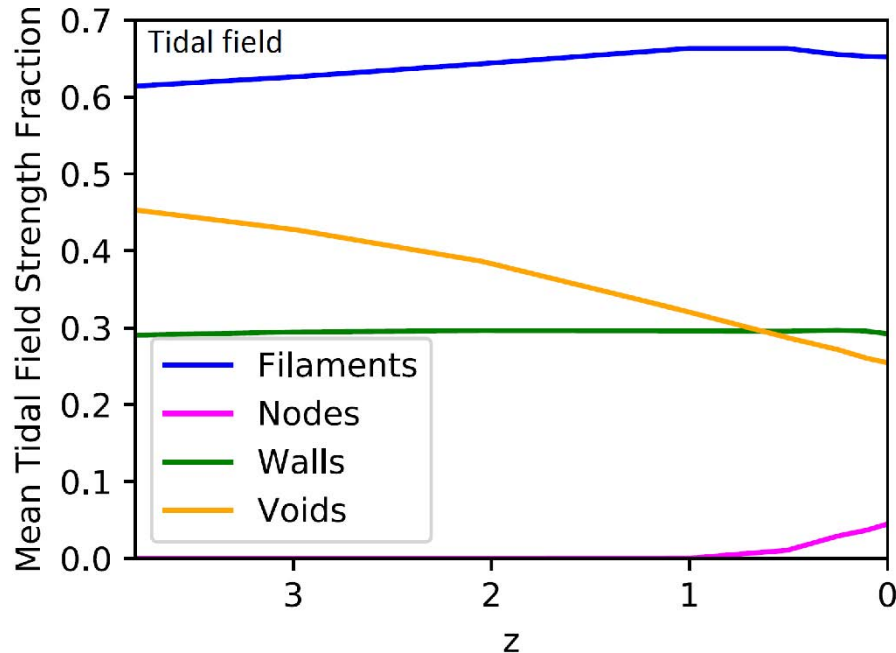


**Voids dominant through organized field:
Principal organizers Cosmic Web network**

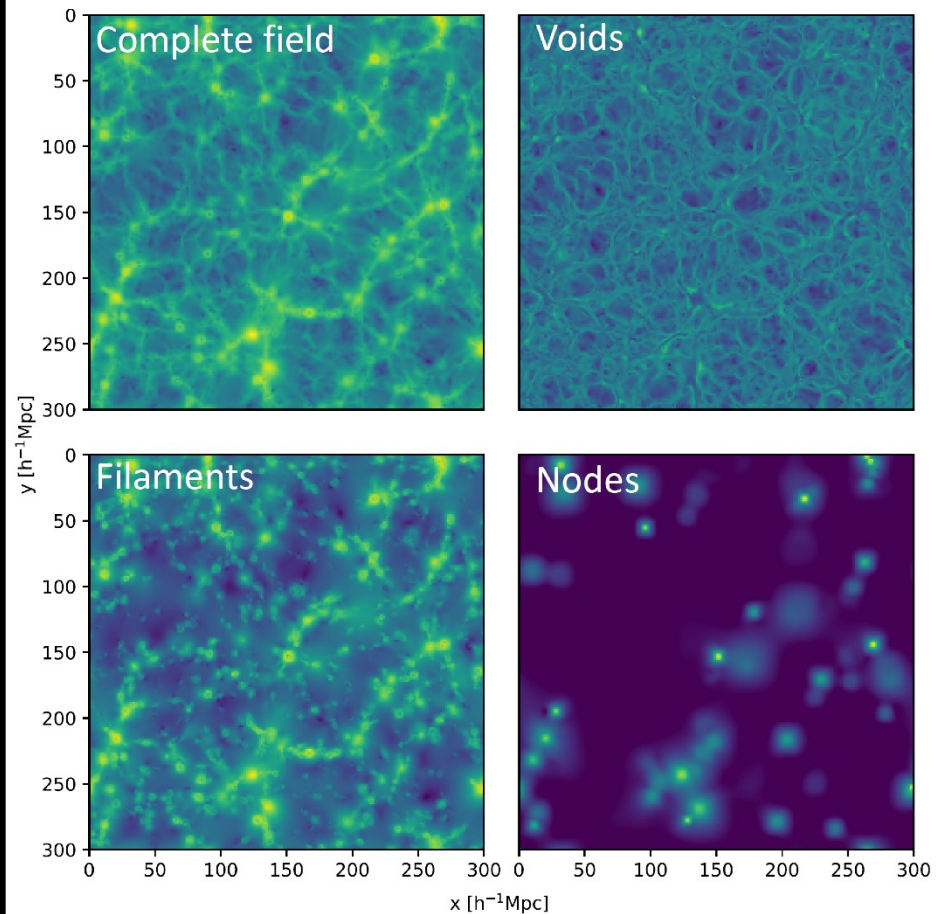


Tidal Field: Evolution

$$T_{ij}(\vec{r}) = \frac{3\Omega_m H_0^2}{8\pi} \int d\vec{r}' \delta_m(\vec{r}' - \vec{r}) \frac{3(\vec{r}'_i - \vec{r}_i)(\vec{r}'_j - \vec{r}_j) - |\vec{r}' - \vec{r}|^2 \delta_{ij}}{|\vec{r}' - \vec{r}|^5} - \frac{1}{2} \Omega H^2 \delta(\vec{r}) \delta_{ij}$$



- Filaments dominant throughout cosmic evolution
- Voids much more influential at high z
- Clusters recently quickly grow more influential



Cosmic Web

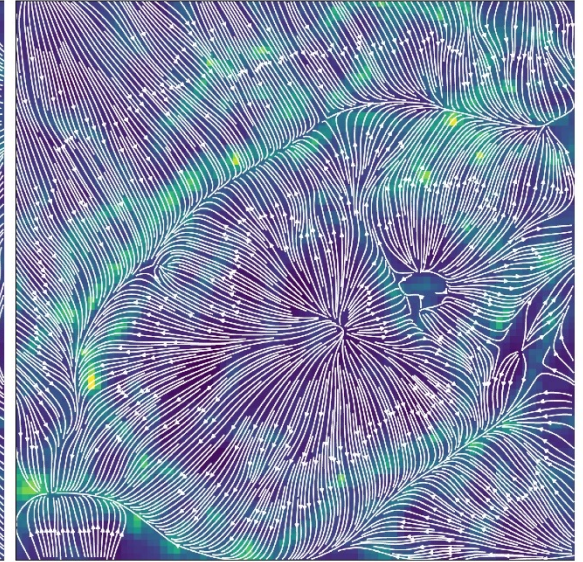
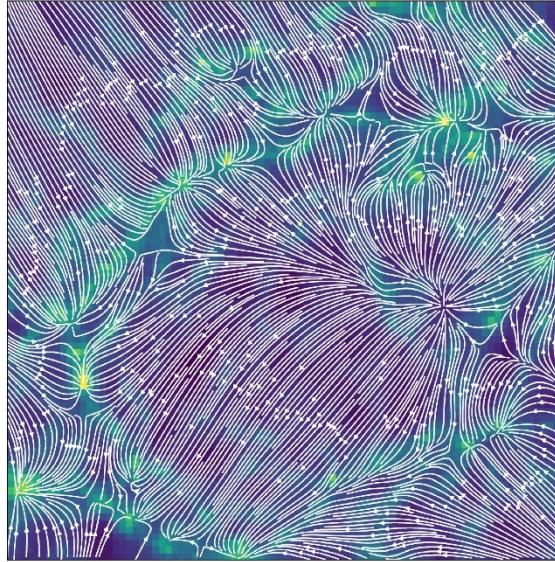
Voids, the organizers

Void Force & Flow field

Complete

Void (induced)

Flowlines

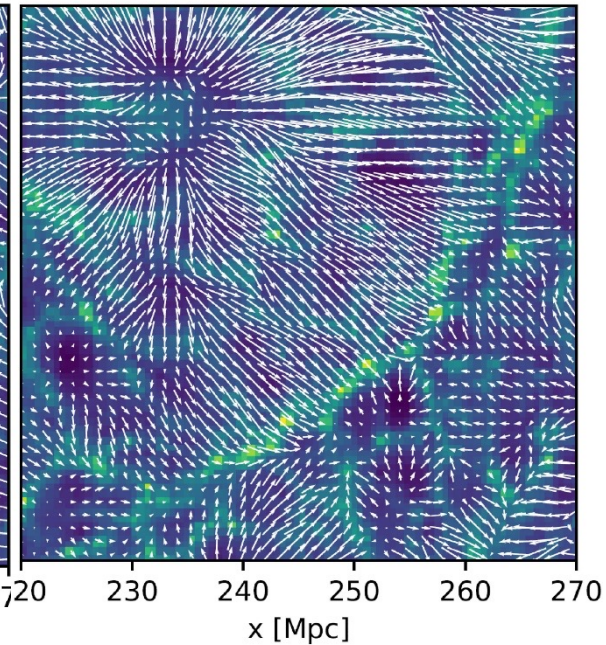
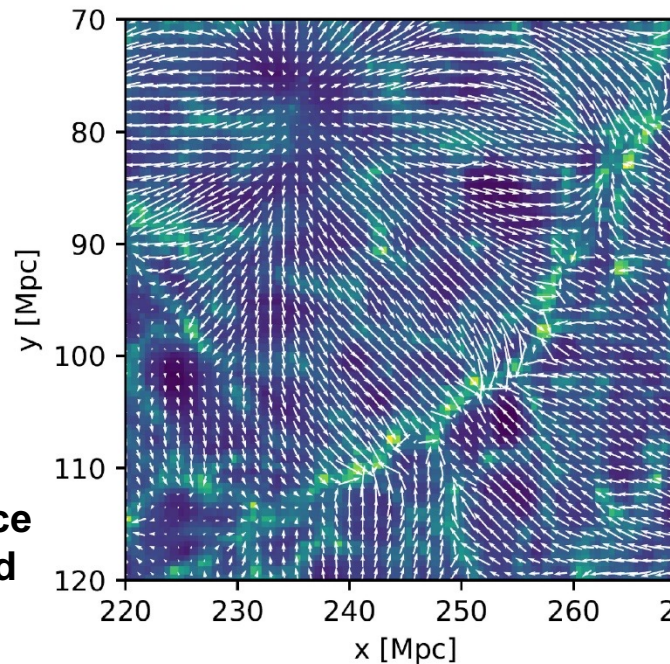


Voids dominate

- Force field
- Velocity flow field

in and around void regions

Force
Field

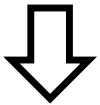


Void Tidal Field

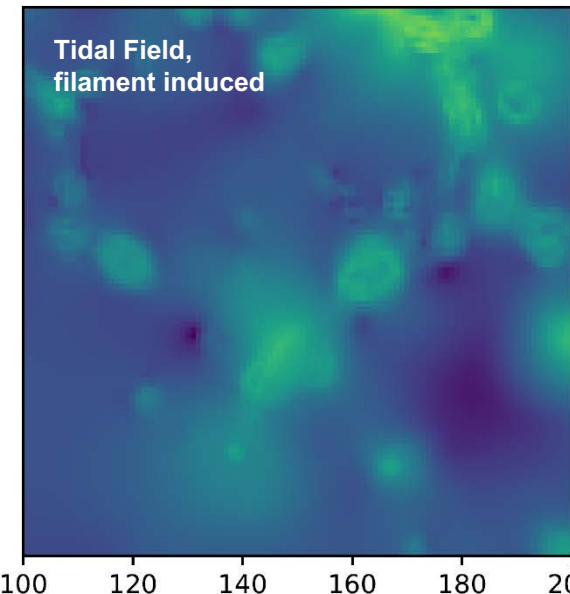
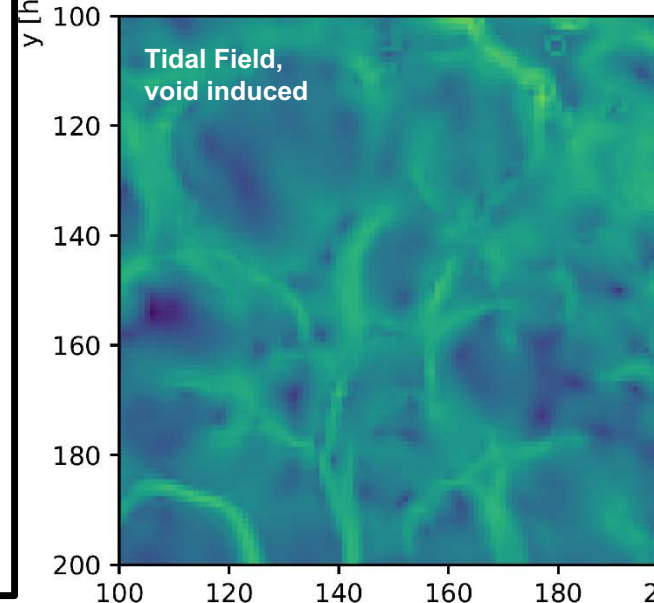
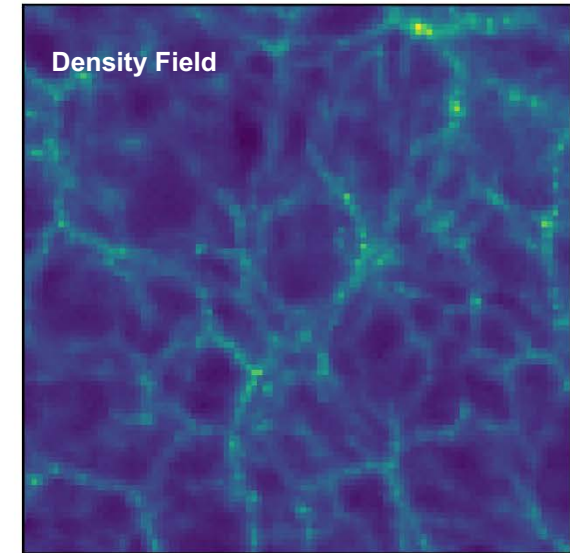
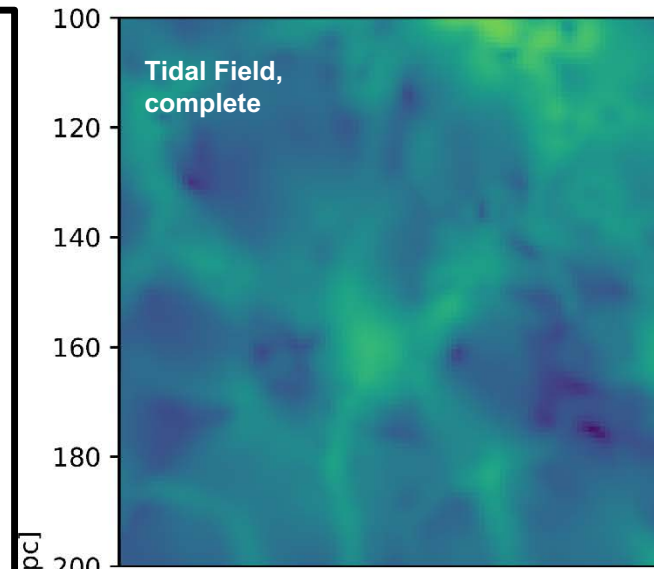
Voids dominate

- Tidal field
spatial structure
- spatial correlation
density field -
void induced tidal field

- Tidal field
agent of Cosmic Web



- Voids organize
Cosmic Web



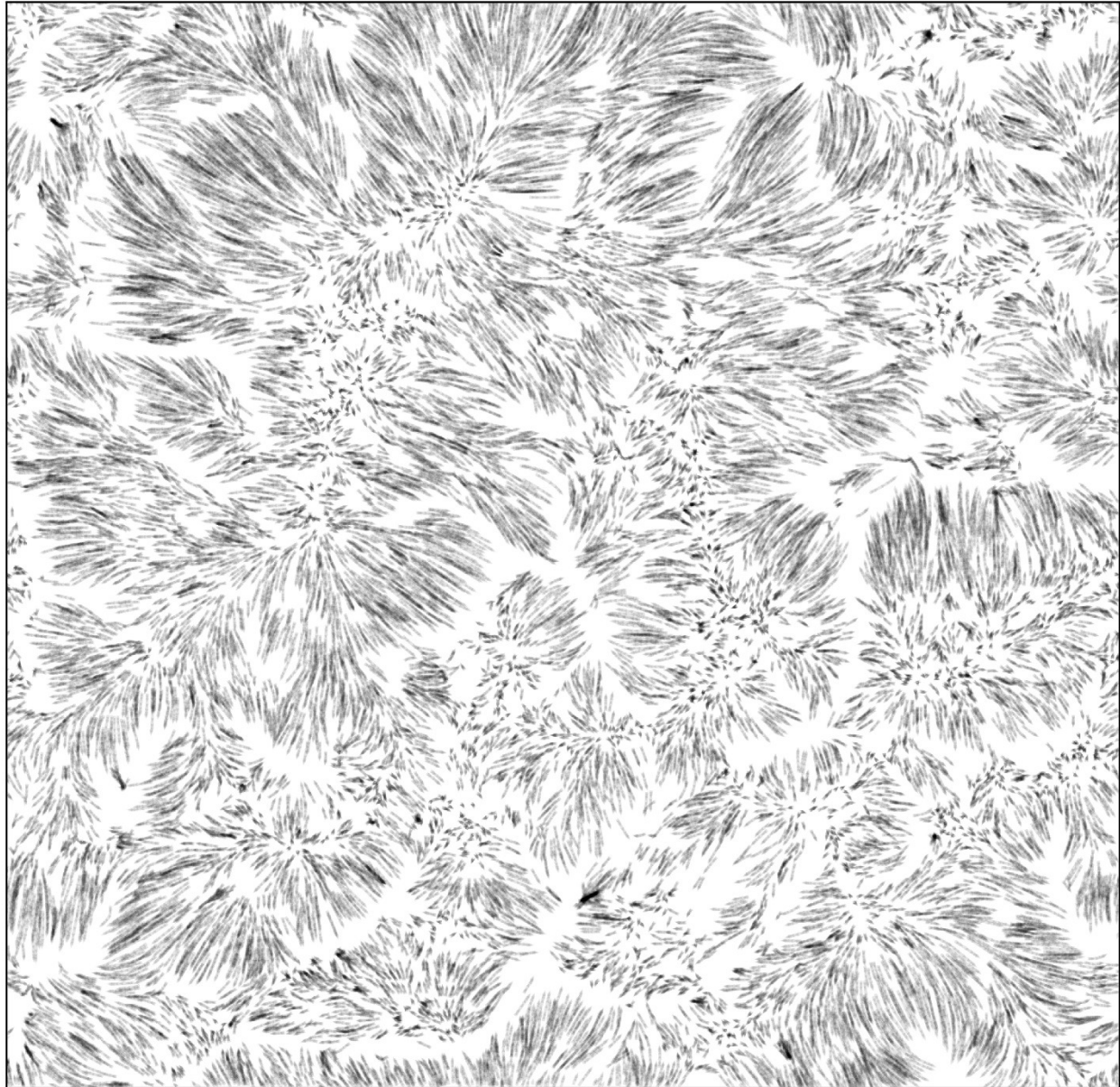
x [h^{-1} Mpc]

Voids: Organizer Cosmic Web

Voids dominate

- Tidal field
spatial structure
 - spatial correlation
density field -
void induced tidal field
 - Tidal field
agent of Cosmic Web
- ↓
- Voids organize
Cosmic Web

Void Force Field

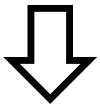


Voids: Organizer Cosmic Web

Voids dominate

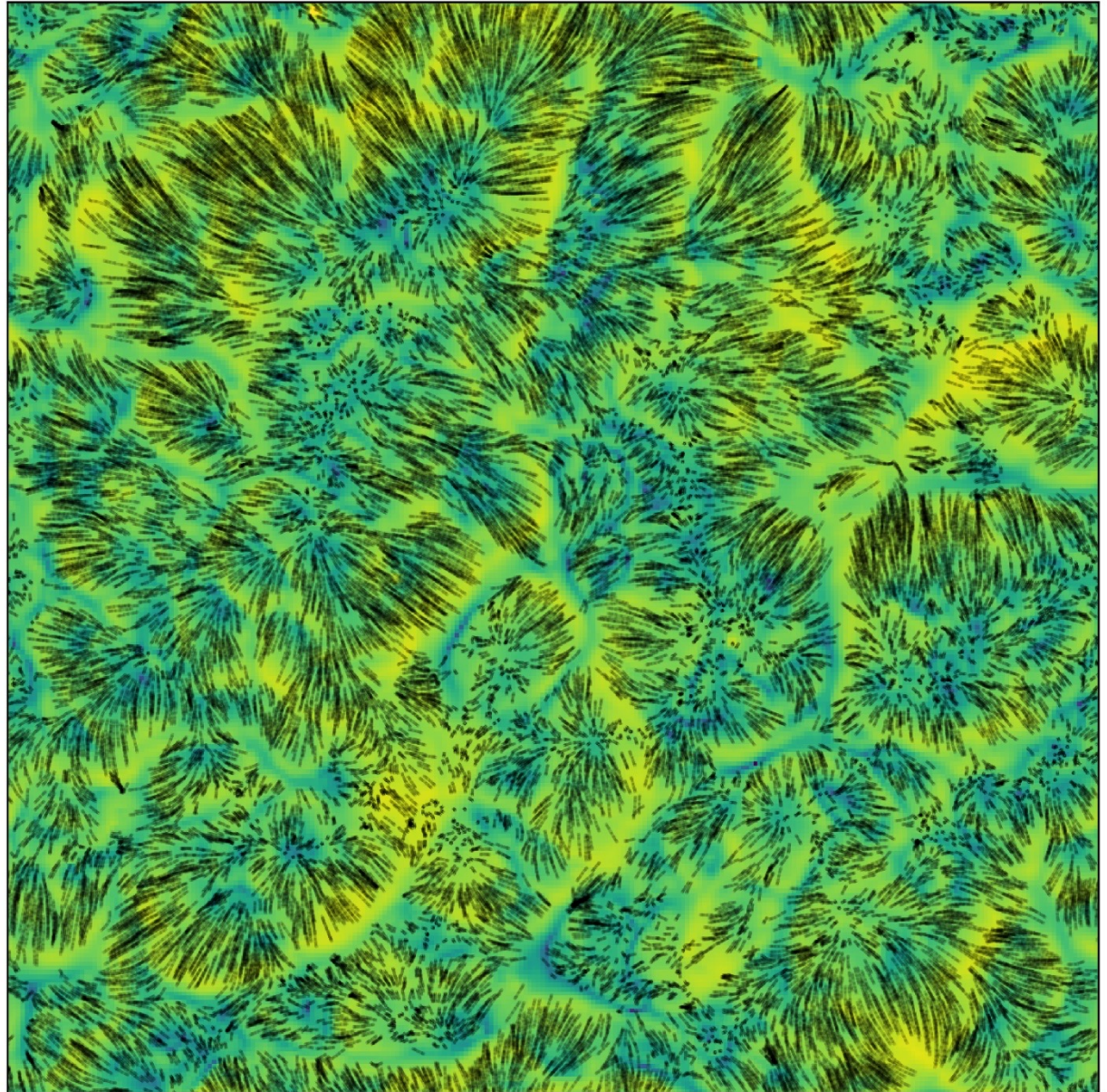
- Tidal field
spatial structure
- spatial correlation
density field -
void induced tidal field

- Tidal field
agent of Cosmic Web



- Voids organize
Cosmic Web

Void outflow: streaklines



Summary

- *Filaments rule!*

yet

- *Voids organize!*

- strict hierarchy
filaments-voids-walls-cluster nodes
- Filaments dominate force & tidal field
- Voids organize:
 - structural pattern cosmic web
 - connectivity cosmic web
- Voids very important at high z
- Cluster nodes:
relatively recent & fast
growing influence

