5^{th} European Actuarial Journal Conference

Programme and abstracts

August 22 - 24, 2022
Tartu

Contents

Welcome to EAJ 2022 in Tartu

On behalf of the Institute of Mathematics and Statistics of University of Tartu and Estonian Actuarial Society, we are delighted to welcome you to the 5th European Actuarial Journal conference in Tartu, Estonia (August 22–24, 2022). The aim of the conference is to bring together practicing actuaries and academics to discuss current topics in the insurance industry and to examine links between actuarial theory and practice.

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Keynote speakers

Annamaria Olivieri

Annamaria Olivieri is Professor of Mathematical Methods for Economics, Actuarial Science and Finance at the Department of Economics and Management of University of Parma (Italy). She is an Associate Investigator of CEPAR, UNSW (Sydney). She is Associate Editor of the European Actuarial Journal and editorial board member of Risks. Her research interests include life and health insurance modelling and actuarial risk management. Her work has been published (among the others) in Insurance: Mathematics and Economics, Astin Bulletin, Annals of Actuarial Science, European Actuarial Journal, Risks.

Martin Eling

Martin Eling is Director of the Institute of Insurance Economics and Professor for Insurance Management at the University of St. Gallen. He is doing empirical research in the intersection of insurance business, economics, and mathematics. His research interests include a broad range of topics including strategic management, financial management, risk management, and asset management. The current agenda includes new research topics such as cyber risk, microinsurance, and long-term care. His research has been awarded by several leading institutions including the American Risk and Insurance Association (ARIA), the Casualty Actuarial Society (CAS), and the National Association of Insurance Commissioners (NAIC). Moreover, he had visiting positions at the University of Torino and Urbino (both in Italy). He is also board member of Concordia, one of Switzerland's largest health insurance companies.

Julien Trufin

Julien Trufin is Associate Professor of Actuarial Science in the Department of Mathematics at the Free University of Brussels (ULB, Belgium). His research interests include risk classification, loss reserving and credibility theory. He is co-editor of the European Actuarial Journal and associate editor of ASTIN Bulletin and Methodology and Computing in Applied Probability. His work is published in (among others) Insurance: Mathematics and Economics, Scandinavian Actuarial Journal, North American Actuarial Journal, ASTIN Bulletin and the European Actuarial Journal.

Gauss prize

To support and motivate especially scientists and young actuaries to investigate open actuarial questions, the German Society for Insurance and Financial Mathematics (DGVFM) and German Association of Actuaries have established the annual Gauss Prize in 1998.

It awards up to three papers with high practical relevance. The aim is to build bridges between actuarial science and the insurance and finance industry. Since 2019 the prize is given out to best contributions to the European Actuarial Journal.

The Gauss Prize 2021 of the DGVFM for the best paper in the EAJ in that calendar year will be awarded to Łukasz Delong, Mathias Lindholm and Mario Wüthrich for their paper *Making Tweedie's compound Poisson model more accessible*. The paper appeared in Vol. 11/1 of the EAJ in open access (https://link.springer.com/article/10.1007/s13385-021-00264-3) and abstract is also given below.

Making Tweedie's compound Poisson model more accessible

Łukasz Delong, Mathias Lindholm and Mario V. Wüthrich

23 Aug
10:00

The most commonly used regression model in general insurance pricing is the compound Poisson model with gamma claim sizes. There are two different parametrizations for this model: the Poisson-gamma parametrization and Tweedie's compound Poisson parametrization. Insurance industry typically prefers the Poisson-gamma parametrization. We review both parametrizations, provide new results that help to lower computational costs for Tweedie's compound Poisson parameter estimation within generalized linear models, and we provide evidence supporting the industry preference for the Poisson-gamma parametrization.

Useful Information

Coffee breaks and lunches are served in the foyer and Delta Café. All the coffe breaks and lunches are included in the conference fee.

Wi-Fi is available at the conference venue. You can use *eduroam* or *ut-public* network.

The free **walking tour** on Monday starts at 17:30 from the conference venue and lasts about 2 hours.

Conference dinner will take place in Barge Hall (Lodjakoda), which is located at about 10 minute walking distance from the conference venue along the picturesque riverside of Emajõgi. Barge Hall serves as a building place of wooden ships and as the exhibition place of their history, it is also an event centre. See more at https://www.lodi.ee/barge-hall or https://arhitektuuripreemiad.ee/en/object/barge-chamber/
The dinner is served in buffet style, recommended dress code: casual and comfortable.

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Platinum sponsor

Swedbank is the largest bank in Estonia as it has more than 850,000 private and 130,000 business customers.

Swedbank offers a wide selection of financial services, including accounts, mortgages, insurances, credit cards, loans, pensions and investments, both for private customers and companies. The goal is to help people and companies keep their finances in order over the long term. Swedbank follows three guiding values in all of their activities: they aspire to be open, simple and caring.

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Eesti Kindlustusseltside Liit

The Estonian Insurance Association (EKsL) is a professional association that unites all of the insurers operating in Estonia.

The EKsL operates as the central insurance organisation that represents the interests of insurers, promotes the development of insurance activities, harmonises the insurance practices of its members and implements cross-market solutions for damage prevention.

Timetable

Sunday, August 21

19:00-22:00	**Welcome reception at the White Hall of the University of Tartu Museum**

Monday, August 22

KL: Keynote Lecture
PS: Parallel session

8:00–17:30	**Registration is open (at info desk)**		
8:30–9:00	**Morning coffee**		
9:00–9:30	**Opening ceremony**		
9:30–10:30 Room 1037	KL	**Martin Eling** University of St. Gallen	Is cyber risk insurable?
10:30–11:00	**Coffee break**		
11:00–12:40	PS1	Room 1019 Room 1020	Mortality modelling Ruin probabilities
12:40–14:00	**Lunch break**		
14:00–15:20	PS2	Room 1019 Room 1020	Advances in life insurance Special types of risk processes
15:20–15:40	**Coffee break**		
15:40–17:20	PS3	Room 1019 Room 1020	Multi-state models in life insurance Measuring risks
17:30	**Walking tour**		

Parallel session 1, Monday 11:00-12:40

Mortality modelling, Room 1019

Chair: Michèle Vanmaele	
Benjamin Roelants du Vivier	Impact of correlation between interest rates and mortality rates on the value of a zero-coupon survival bond
Jens Robben	Dealing with mortality shocks in a stochastic multi-population mortality model
Alaric J.A. Müller	Joint lifetime modelling with mIPH distributions
Peter Hieber	Mortality credits within large survivor funds

Ruin probabilities, Room 1020

Chair: Alfredo D. Egídio dos Reis	
Stéphane Loisel	Optimal prevention strategies in the classical risk model
Jonas Sprindys	The Gerber-Shiu discounted penalty function for the bi-seasonal discrete time risk model
Naoyuki Ishimura	Efficient numerical computation of the ruin probability
Jonas Šiaulys	Martingale approach to derive Lundberg-type inequalities
Artur Sepp	Modeling implied volatility surfaces of crypto options

Parallel session 2, Monday 14:00-15:20

Advances in life insurance, Room 1019

Chair: Griselda Deelstra	
Jinbo Zhao	A model for multifactorial genetic disorders to quantify the impact of polygenic risk scores on lifeinsurance: a simulation based model using heartattack as a case study
Jannes Tjark Rastedt	Actuarial calculations for reserve-dependent payments in life insurance under information shrinkage
Julian Jetses	A general surplus decomposition principle in life insurance
Jennifer Alonso-García	A hybrid variable annuity contract embedded with living and death benefit riders

Special types of risk processes, Room 1020

Chair: Martin Eling	
Rui Cardoso	On a penalty function in the Erlang renewal dual risk model under independent randomised observations
Dina Finger	Ruin and profitability in Bitcoin mining: analysis of pools and empirical evidence
Jose Miguel Flores-Contró	On a risk process with deterministic investment and multiplicative jumps - an application to poverty trapping
Jose Miguel Flores-Contró	The role of direct cash transfers towards extreme poverty alleviation - an Omega risk process

Parallel session 3, Monday 15:40-17:20

Multi-state models in life insurance, Room 1019

Chair: Jennifer Alonso-García	
Jamaal Ahmad	Statistical inference in an aggregated Markov chain model in life insurance
Theis Bathke	Two-dimensional forward and backward transition rates
Oscar Peralta	Homogeneous approximations of time-inhomogeneous semi-Markov life insurance models
Oliver Lunding Sandqvist	IBNR and RBNS models in multi-state life insurance
Christian Furrer	Extension of as-if-Markov modeling to scaled payments

Measuring risks, Room 1020

Chair: Stéphane Loisel	
Rodrigue Kazzi	Assessing model uncertainty for log-symmetric distributions
Christian Laudagé	Combining multi-asset and intrinsic risk measures
Saulius Paukštys	Tails of moments of sums with heavy-tailed summands and applications to the Haezendonck-Goovaerts risk measure
Gero Junike	Representation of concave distortions and applications
Hang Nguyen	Calculation of risk measures of variable annuity portfolios using neural network

Tuesday, August 23

8:30–14:00	**Registration is open (at info desk)**		
8:30–9:00	**Morning coffee**		
9:00–10:00 Room 1037	KL	**Annamaria Olivieri** University of Parma	Flexibility in annuity benefits in view of mortality/longevity uncertainty and individuals' longevity risk appetite (or unawareness)
10:00–10:30 Room 1037	**Gauss prize/EAJ 2021 best paper session***		
10:30–11:00	**Coffee break**		
11:00–12:40	PS4	Room 1019 Room 1020	Pensions Risk management and mitigation
12:40–14:00	**Lunch break**		
14:00–15:40	PS5	Room 1019 Room 1020	Longevity Dependence modelling
15:40–16:00	**Coffee break**		
16:00–18:00	PS6	Room 1019 Room 1020	Advances in reinsurance and related areas Valuation and reserving
19:00–23:00	**Conference dinner at Barge Hall**		

*__Gauss prize/EAJ 2021 best paper session:__
Łukasz Delong, Mathias Lindholm and Mario V. Wüthrich
Making Tweedie's compound Poisson model more accessible,
presented by Łukasz Delong and Mathias Lindholm.

Parallel session 4, Tuesday 11:00-12:40

Pensions, Room 1019

Chair: Jaanus Sibul	
Ivan Alexis Fonseca Diaz	Optimal multiperiod mixture between pay-as-you-go and funded financial systems for social security
Onofre Simões	Development of a tool for making projections in DC pension schemes
Abraham Hernández-Pacheco	A probability of ruin approach to optimize pension fund investments
Jennifer Alonso-García	Public pension schemes – intergenerational risk sharing
Marlene Koch	Mandatory pension saving and homeownership

Risk management and mitigation, Room 1020

Chair: Tim Boonen	
Gabriela Zeller	Optimal price structure of cyber insurance policies with risk mitigation services
Griselda Deelstra	A multi-curve HJM factor model for pricing and risk management
Tachfine El Alami	Risk aggregation under IFRS 17: An ultimate run-off adaptation of Solvency 2 elliptic aggregation
Karim Barigou	Surveillance of actuarial assumptions in the Enterprise Risk Management framework
Roberto Carcache Flores	A new approach to Markowitz portfolio optimization using ruin-based outcomes

Parallel session 5, Tuesday 14:00-15:40

Longevity, Room 1019

Chair: Annamaria Olivieri	
Michèle Vanmaele	Mortality/longevity risk-minimization with or without securitization
Indradeb Chatterjee	Social welfare under restricted risk classification
Tahir Choulli	How mortality and/or longevity risks impact log-optimal portfolio?
Wojciech Otto	Model of lifetable evolution Part 1: Extracting calendar and cohort effects form the data
Wojciech Otto	Model of lifetable evolution Part 2: Time series model with stochastically varying drift

Dependence modelling, Room 1020

Chair: Tõnu Kollo	
Martynas Manstavičius	Diversity of bivariate concordance measures
Christopher Blier-Wong	Exchangeable FGM copulas
Christopher Blier-Wong	Micro-level collective risk models under FGM dependence
Alfredo D. Egídio dos Reis	Risk model with dependent frequency and severity, premium and ruin probability calculation
Jorge Yslas	Phase-type mixture-of-experts regression for loss severities

Parallel session 6, Tuesday 16:00-18:00

Advances in reinsurance and related areas, Room 1019

Chair: Kalev Pärna	
Tim J Boonen	Revisiting Arrow's problem in a sequential game
Yevhen Havrylenko	Risk-sharing in equity-linked insurance products: Stackelberg equilibrium between an insurer and a reinsurer
Alexandra Bugalho de Moura	On the impact of dependences and constraints in the optimal reinsurance treaty
Thijs Kamma	Dual formulation of the optimal consumption problem with multiplicative habit formation
Romain Gauchon	On the use of convolution order for expected utility differentiation and optimization
Eva Verschueren	Red light, green light: gaussian process regression to validate capped volatility swaps

Valuation and reserving, Room 1020

Chair: Boualem Djehiche	
Marcin Szatkowski	One-year and ultimate reserve risk in Mack Chain Ladder model
Henning Zakrisson	A collective reserving model with claim openness
Mathias Lindholm	Bias regularisation, dispersion modelling, and auto-tariffication
Filip Lindskog	Multiple-prior valuation of cash flows subject to capital requirements
Nils Engler	Convergence and robustness to model uncertainty in multiple period valuation problems
Lorenzo Marchi	On fair pricing via regularization

Wednesday, August 24

8:30–12:00	**Info desk is open**		
8:30– 9:00	**Morning coffee**		
9:00–10:00 Room 1037	KL	**Julien Trufin** Free University of Brussels	Non-life insurance pricing: boosting trees and diagnostic tools to compare competing models
10:00–10:30	**Coffee break**		
10:30–12:10	PS7	Room 1019	Long-term care
		Room 1020	Machine learning methods in insurance
12:15–12:30 Room 1037	**Closing ceremony**		
12:30–14:00	**Lunch**		

Parallel session 7, Wednesday 10:30-12:10

Long-term care, Room 1019

Chair: Peter Hieber	
Aleksandr Shemendyuk	Study of institutionalized elderly profiles derived from multiple health factors
Aleksandr Shemendyuk	Determinants of institutional long-term care of dependent elderly in Switzerland
Andrey Ugarte	On the drivers of potential customers' interest in long-term care insurance: evidence from Switzerland
Andrey Ugarte	Are long-term care insurance customers likely to come with greater risks for insurers? Reflections and simulations from a survey in Switzerland

Machine learning methods in insurance, Room 1020

Chair: Julien Trufin	
Łukasz Delong	The use of autoencoders for training deep neural networks with mixed categorical and numerical features
Freek Holvoet	Neural networks for frequency-severity modelling: a benchmark study from data preprocessing steps to technical tariff
Bernard Wong	Ensemble distributional forecasting for insurance loss reserving
Lina Palmborg	Premium control with reinforcement learning
Bavo DC Campo	Insurance fraud network data simulation machine: Synthetic data sets and fraud detection strategies

List of Abstracts

Is cyber risk insurable?

Martin Eling

University of St. Gallen, Switzerland, martin.eling@unisg.ch

KL

22 Aug
9:30
Room 1037

We use data on cyber losses and model the potential market equilibrium for cyber insurance to document a market failure when portfolios of cyber risk are constructed. This can be explained by the distinct properties of cyber risk, namely heavy tails, strong (tail) dependencies, high costs due to asymmetric information, and modelling risk. Our results help to explain why many insurance companies are reluctant to offer cyber insurance on a broad scale. We also discuss ways to overcome the market failure. Our analyses expand results from catastrophic insurance and have implications for corporate risk management and public policy.

Flexibility in annuity benefits in view of mortality/longevity uncertainty and individuals' longevity risk appetite (or unawareness)

KL

23 Aug
9:00
Room 1037

Annamaria Olivieri

University of Parma, Italy, annamaria.olivieri@unipr.it

Despite the growing need for individuals to protect their assets from the longevity risk to which they will be exposed after retirement, the demand for products offering longevity guarantees, namely annuities, remains low. Possible reasons include the cost of guarantees, but also the rigidity of the traditional annuity design, which implies an irreversible decision at issue. This contrasts with the attitude of individuals who, because of their risk appetite or unawareness, seem to prefer to postpone final decisions in this regard. This way, they show a willingness to retain their longevity risk up to some (high) age. On the other hand, traditional annuity designs expose the provider to the aggregate longevity risk.

Flexibility in the annuity design could, at least partially, reconcile the different needs of individuals and providers. Flexibility can be introduced in different ways. We address two solutions. First, the guarantees can be relaxed, and the related costs can be reduced, by allowing the benefit amount to fluctuate (up or down) according to a given mortality/longevity experience. Second, instead of an upfront loading at issue, periodic fees could be adopted, as they are more suitable to support a revision of the arrangement after issue. While the literature includes already several contributions on mortality/longevity-linked benefits, a pricing structure based on periodic fees has received little attention so far. We develop a discussion in this respect, considering mortality/longevity-linked annuities.

Non-life insurance pricing: boosting trees and diagnostic tools to compare competing models

KL

24 Aug
9:00 Room
1037

Julien Trufin

Free University of Brussels, Belgium, julien.trufin@ulb.be

Thanks to its outstanding performances, boosting has rapidly gained wide acceptance among actuaries. To speed up calculations, boosting is often applied to gradients of the loss function, not to responses (hence the name gradient boosting). When the model is trained by minimizing Poisson deviance, this amounts to apply the least-squares principle to raw residuals. This exposes gradient boosting to the same problems that lead to replace least-squares with Poisson Generalized LinearModels (GLM) to analyze low counts (typically, the number of reported claims at policy level in lines). In a first time, this talk shows that boosting can be conducted directly on the response under Tweedie loss function and log-link, by adapting the weights at each step.

Modern data science tools are effective to produce predictions that strongly correlate with responses. Model comparison can therefore be based on the strength of dependence between responses and their predictions. Positive expectation dependence turns out to be attractive in that respect. In a second time, the present talk proposes an effective testing procedure for this dependence concept and applies it to compare two models. Empirical illustrations using insurance loss data demonstrate the relevance of the approach for model selection in supervised learning. The most positively expectation dependent predictor can then be autocalibrated to obtain its balance-corrected version that appears to be optimal with respect to Bregman, or forecast dominance. Under autocalibration, it is shown that Lorenz curve and concentration curve coincide and that the integral of the concentration curve is equivalent to Gini coefficient.

Statistical inference in an aggregated Markov chain model in life insurance

22 Aug
PS3

Jamaal Ahmad[1] **and Mogens Bladt**[2]

[1]*University of Copenhagen, Denmark, jamaal@math.ku.dk*
[2]*University of Copenhagen, Denmark, bladt@math.ku.dk*

Keywords: multi-state life insurance, aggregated Markov chains, inhomogeneous phase-type distributions, Markov chain Monte Carlo methods, EM algorithms

In multi-state life insurance, the random pattern of states of the insured is usually assumed to follow a time-inhomogeneous Markov jump process on a finite state space. In these kinds of models, there exists a wide variety of well-known methods for statistical inference based on data of the trajectories of the states. In the context of phase-type distributions, this setting leads to sojourn times in the different states that follows one-dimensional inhomogeneous phase-type distributions, cf. [1].

In this talk, we introduce a so-called aggregated Markov chain model in life insurance, where we assign a number of micro states to each biometric macro state, leading to sojourn times in macro states following inhomogeneous phase-type distributions of general dimension. Since the observable data is the trajectories of the macro states only, this extension leads to incomplete data. By assuming piecewise constant transition rates on micro level, we develop algorithms for both parametric and non-parametric estimation of said transition rates using EM algorithms and Markov chain Monte Carlo methods (MCMC), respectively, extending the work of [2, 3]. The talk ends with an application of the algorithms on simulated data from a time-inhomogeneous semi-Markov model as that of [4], which serves to illustrate possibilities and limitations of our extended model in actuarial applications.

References

[1] Albrecher, H., Bladt, M. (2019). Inhomogeneous phase-type distributions and heavy tails. *Journal of Applied Probability* 56(4), 1044–1064.

[2] Albrecher, H., Bladt, M., Yslas, J. (2022). Fitting inhomogeneous phase-type distributions to data: the univariate and the multivariate case. *Scandinavian Journal of Statistics* 49(1), 44–77.

[3] Bladt, M., Gonzalez, A., Lauritzen, S.L. (2003). The estimation of phase-type related functionals using Markov chain Monte Carlo methods. *Scandinavian Actuarial Journal* 2003(4), 280–300.

[4] Hoem, J.M. (1972). Inhomogeneous semi-Markov processes, select actuarial tables, and duration-dependence in demography. In T.N.E. Greville, editor, *Population Dynamics*, pages 251–296. Academic Press, New York.

Public pension schemes – intergenerational risk sharing

Hélène Morsomme[1], **Jennifer Alonso-García**[2] **and Pierre Devolder**[3]

23 Aug
PS4

[1]*Université catholique de Louvain, Belgium, helene.morsomme@uclouvain.be*
[2]*Universite Libre De Bruxelles, Belgium, CEPAR, Netspar, jennifer.alonso.garcia@ulb.be*
[3]*Université catholique de Louvain, Belgium, pierre.devolder@uclouvain.be*

Keywords: public pension, intergenerational risk sharing, risk analysis, hybrid scheme, automatic balance mechanism

Population ageing undermines the traditional social security pension system. Most European countries have a pay-as-you-go (PAYG) and defined benefit (DB) public pension system. In this context, guaranteed benefits could hardly be held and the demographic risk due to ageing would be entirely borne by an increase of the contributions paid by the active workers [1, 3].

To avoid a dramatic increase of the contributions and in order to maintain simultaneously the financial sustainability and the social adequacy of the public pension system, risk sharing of the demographic risk between workers and different retired generations seems adequate. Proposed system is hybrid – intermediate plan between DB and DC (defined contribution) schemes – and includes an automatic balance mechanism (ABM) trough a simultaneous adaptation of the contribution rate and the replacement rate [2, 4].

Our purpose is to propose a risk sharing model based on intergenerational optimisation. We apply the proposed model on data of the projection of the Belgian population and we develop a stochastic analysis of the benefits and the contributions for specific scenarios: the extreme DB and DC schemes and an intermediate scheme with the Musgrave rule – constant replacement rate net of contributions.

References

[1] Alonso-García, J., Boado-Penas, M. D. C., Devolder, P. (2018). Adequacy, fairness and sustainability of pay-as-you-go-pension-systems: defined benefit versus defined contribution. *The European Journal of Finance* 24(13), 1100-1122.

[2] Cairns, A. (2000). Some notes on the dynamics and optimal control of stochastic pension fund models in continuous time. *ASTIN Bulletin: The Journal of the IAA* 30(1), 19-55.

[3] Godínez-Olivares, H., del Carmen Boado-Penas, M., Haberman, S. (2016). Optimal strategies for pay-as-you-go pension finance: A sustainability framework. *Insurance: Mathematics and Economics* 69, 117-126.

[4] Musgrave, R. A. (1981). A reappraisal of financing social security. *Social security financing* 89, 109-20.

Risk model with dependent frequency and severity, premium and ruin probability calculation

23 Aug
PS5

Renata G. Alcoforado[1,2] **and Alfredo D. Egídio dos Reis**[1]

[1]*Universidade de Lisboa, Portugal, alfredo@iseg.ulisboa.pt*
[2]*Universidade Federal de Pernambuco, Brazil, alcoforado.renata@ufpe.br*

Keywords: dependent risks, insurance risk model, premium calculation, ruin probability, real data application

A common assumption made in classical risk theory with application in insurance modelling is that the “claim counts" and the “claim severity" are independent. It may not be the case in many situations. In recent times there have been authors working with models allowing some sort of dependence, good examples with application in motor insurance are [4], [3] and [6]. Also, we mention [1], [2] and [5]. They use different methods to capture dependence, eg. the first and last use copulae and the second uses GLM.

By using using real data from two different, but related, branches on housing and liability insurance from an anonymous insurer, we work numerical illustrations. We consider dependence between claim counts and severity of claims, as well as among individual claims in order to estimate subsequent premiums and corresponding ruin probabilities.

We work formulae and numerical results for both ruin probabilities and adjusted premiums in some sort of models. We start from studying two separate compound Poisson models and study existence of dependence, then its size using appropriate methods, either copulae, regression or mixing Poisson parametrization. Our database has the novelty of not coming from automobile insurance, thus bringing other challenges.

References

[1] Albrecher, H., Constantinescu, C., and Loisel, S. (2011). Explicit ruin formulas for models with dependence among risks. *Insurance: Mathematics and Economics* 48(2), 265–270.

[2] Andrade e Silva, J. and Centeno, L. (2017). Ratemaking of Dependent Risks. *ASTIN Bulletin* 47(3), 875–894.

[3] Garrido, J., Genest, C., and Schulz, J. (2016). Generalized linear models for dependent frequency and severity of insurance claims. *Insurance: Mathematics and Economics* 70, 205–215.

[4] Li, B., Ni, W., and Constantinescu, C. (2015). Risk models with premiums adjusted to claims number. *Insurance : Mathematics and Economics* 65(2015), 94–102.

[5] Moura, A. and Centeno, L. (2020). Optimal Reinsurance of Dependent Risks. *Forthcoming: REVSTAT, https://www.ine.pt/revstat/pdf/OptimalReinsuranceofDependentRisks.pdf*.

[6] Ni, W., Constantinescu, C., Egidio dos Reis, A. D., and Maume-Deschamps, V. (2020). Pricing foreseeable and unforeseeable risks in insurance portfolios. *Working Papers, https://arxiv.org/pdf/2008.03123.pdf*.

Surveillance of actuarial assumptions in the Enterprise Risk Management framework

23 Aug
PS4

Karim Barigou[1], **Nicole El Karoui**[2], **Stéphane Loisel**[1] **and Yahia Salhi**[1]

[1]*Univ Lyon, Université Claude Bernard Lyon 1, karim.barigou@univ-lyon1.fr*
[2]*LPMA, UMR CNRS 7599 Université Paris 6, France*

Keywords: monitoring, change-point detection, Poisson process, longevity risk, risk management

Recently there has been a growing interest in both the monitoring of insurance processes (e.g. the number of death tolls due to Covid-19) and Enterprise Risk management, but not on the interplay between monitoring and risk management. Even though there is a rich literature in statistical process control, often simple moving averages are used in practice. In this paper, we show how the CUSUM, one of the most popular sequential change-point detection procedures, can be efficiently used to detect an increase or decrease in the intensity of a counting process (see [1]) within the risk management framework. Several case studies are considered both in life (mortality and longevity risks) and non-life (automobile insurance). In particular, we discuss the connection between the CUSUM and the risk appetite, and show that the CUSUM appears as an effective key risk indicator and early warning of financial distress and declining profitability.

References

[1] El Karoui, N., Loisel, S., and Salhi, Y. (2017). Minimax optimality in robust detection of a disorder time in doubly-stochastic Poisson processes. *The Annals of Applied Probability* 27(4), 2515-2538.

Two-dimensional forward and backward transition rates

Theis Bathke

Carl von Ossietzky Universität Oldenburg, Germany, theis.bathke@uol.de

22 Aug
PS3

Keywords: life and health insurance, non-Markov modelling, prospective and retrospective reserves, second-order moments, free-policy option

Forward transition rates were originally introduced with the aim to evaluate life insurance liabilities market-consistently. While this idea turned out to have its limitations, recent literature repurposes forward transition rates as a tool for avoiding Markov assumptions in the calculation of life insurance reserves, see [1] and [2]. While life insurance reserves are some form of conditional first-order moments, the calculation of conditional second-order moments needs an extension of the forward transition rate concept from one dimension to two dimensions. Two-dimensional forward transition rates are also needed for the calculation of path-dependent life insurance cash-flows as they occur upon contract modifications. Forward transition rates are designed for doing prospective calculations, and by a time-symmetric definition of so-called backward transition rates one can do retrospective calculations.

References

[1] Christiansen, M. C. (2021). On the calculation of prospective and retrospective reserves in non-Markov models. *European Actuarial Journal* 11, 441–462.

[2] Christiansen, M. C., Furrer, C. (2022). Extension of as-if-Markov modeling to scaled payments. https://furrer.dk/christian/christiansenfurrermeasure14032022.pdf.

Micro-level collective risk models under FGM dependence

Christopher Blier-Wong[1], **Hélène Cossette**[2] **and Etienne Marceau**[3]

23 Aug
PS5

[1]*Université Laval, Canada, christopher.blier-wong.1@ulaval.ca*
[2]*Université Laval, Canada, helene.cossette@act.ulaval.ca*
[3]*Université Laval, Canada, etienne.marceau@act.ulaval.ca*

Keywords: collective risk models, FGM copula, stochastic representation, mixed Erlang distribution

Collective risk models, in which the aggregate claim amount of a portfolio is defined as a sum of a random number (frequency) of random claim amounts (severities), play a crucial role. In these models, the classical approach is to assume that the random number of claims and their amounts are independent, even if this might not always be the case [2, 3]. We consider a class of collective risk models, in which the dependence structure of the random number of claims and the individual claim amounts is defined in terms of a multivariate Farlie-Gumbel-Morgenstern (FGM) copula. By leveraging a one-to-one correspondence between the family of FGM copulas and the family of multivariate symmetric Bernoulli random vectors [1], we find closed-form expressions for the moments and Laplace-Stieltjes transform of the aggregate claim amount. We examine the dependence properties of the proposed class of collective risk models. Even if the Farlie-Gumbel-Morgenstern copula may only induce moderate dependence, we show through numerical examples that the cumulative effect of dependence can generate large ranges of values for the expectation, the variance, and risk measures (such as the Tail-Value-at-Risk) of the aggregate claim amount. We present applications of the proposed class of collective risk models in various contexts of non-life insurance.

References

[1] Blier-Wong, C., Cossette, H., Marceau, E. (2022) Stochastic representation of FGM copulas using multivariate Bernoulli random variables. To appear in *Computational Statistics and Data Analysis*.

[2] Czado, C., Kastenmeier, R., Brechmann, E. C., Min, A. (2012). A mixed copula model for insurance claims and claim sizes. *Scandinavian Actuarial Journal* 2012(4), 278-305.

[3] Frees, E. W., Gao, J., Rosenberg, M. A. (2011). Predicting the frequency and amount of health care expenditures. *North American Actuarial Journal* 15(3), 377-392.

Revisiting Arrow's problem in a sequential game

Tim J. Boonen[1] **and Wenjun Jiang**[2]

23 Aug
PS6

[1]*University of Amsterdam, The Netherlands, t.j.boonen@uva.nl*
[2]*University of Calgary, Canada, wenjun.jiang@ucalgary.ca*

Keywords: optimal reinsurance, sequential games, stop-loss reinsurance, pricing

This paper revisits Arrow's classical optimal reinsurance problem in a sequential game. The insurer maximizes expected utility. Under the expectation premium principle, the insurer selects the optimal indemnity function in response to the safety loading factor set by the reinsurer. Knowing the insurer's choice of indemnity function, the reinsurer adjusts the safety loading factor to maximize its expected net profit. We derive the stop-loss structure as solution to this game. Moreover, we also show that the solution to such a sequential game is Pareto optimal in the sense that no party's interest could be improved further without harming the other party's interest. This verifies the efficiency of the sequential game. Some numerical examples are presented to demonstrate the main results.

Insurance fraud network data simulation machine: Synthetic data sets and fraud detection strategies

24 Aug
PS7

Bavo DC Campo[1] **and Katrien Antonio**[1,2]

[1]*KU Leuven, Belgium, bavo.decock@kuleuven.be*
[2]*University of Amsterdam, The Netherlands, katrien.antonio@kuleuven.be*

Keywords: social networks, simulation machine, fraud detection, class imbalance, missing data

Traditionally, the detection of fraudulent insurance claims relies on business rules and expert judgement which makes it a time-consuming and expensive process [1]. Consequently, researchers have been examining ways to develop an efficient and accurate fraud detection model. The use of features engineered from the social network of parties involved in a claim is a particularly promising strategy to flag potentially fraudulent claims (see for example [1, 2]). When developing a fraud detection model, however, we are confronted with several challenges. The uncommon nature of fraud, for example, creates a high class imbalance which complicates the development of analytic classification models. In addition, only a small number of claims are investigated and get a label, which results in a large corpus of unlabeled data. We design a simulation machine that is inspired by the real non-life motor insurance data used in Óskarsdóttir et al. [1]. This data contains both traditional claims characteristics as well as social network features. When generating the synthetic data, the user has control over several data-generating mechanisms. We can specify the total number of policyholders and parties, the desired level of imbalance and the (effect size of the) features in the fraud generating model. Hereby, it enables researchers and practitioners to examine several methodological challenges as well as to back-test their (development strategy of) insurance fraud detection models in a range of different settings.

References

[1] Óskarsdóttir, M., Ahmed, W., Antonio, K., Baesens, B., Dendievel, R., Donas, T., Reynkens, T. (2021). Social Network Analytics for Supervised Fraud Detection in Insurance. *Risk analysis* Advance online publication. doi: 10.1111/risa.13693.

[2] Van Vlasselaer, V., Eliassi-Rad, T., Akoglu, L., Snoeck, M., Baesens, B. (2016). Gotcha! Network-based fraud detection for social security fraud. *Management Science* 63 (9), 3090–3110.

A new approach to Markowitz portfolio optimization using ruin-based outcomes

23 Aug
PS4

Abraham Hernández-Pacheco[1], **Roberto Carcache Flores**[2]

[1]*ISEG & CEMAPRE, Universidade de Lisboa, abe@vitalis.com.mx*
[2]*VITALIS Expertos en Pensiones, Lisboa, rcarcache@vitalis.com.mx*

Keywords: minimum ruin portfolio, modern portfolio theory, ruin theory

Traditional Markowitz portfolio optimization selects portfolios based on criteria related to an investor's risk appetite, ultimately measured only with a portfolio's expected return and variance. This work proposes a new approach which incorporates ruin-based outcomes into the Markowitz framework. This approach uses copula simulations to model portfolios located along the Efficient Frontier, for a given basket of assets. A ruin-based outcome is programmed into the simulation, measuring the probability of a portfolio falling below a predetermined threshold or ruin level.

Thus, the resulting optimal portfolio is Markowitz efficient and minimizes the probability of ruin. We will call this the Minimum Ruin Probability (MRP) portfolio. The results obtained suggest the MRP portfolios fall between those with minimum variance and those with the maximum Sharpe ratio, demonstrating the complex dynamics of volatility and returns that minimize ruin outcomes. Furthermore, the results have also shown parametric sensitivity, suggesting this framework is also flexible for different applications.

On a penalty function in the Erlang renewal dual risk model under independent randomised observations

Renata G. Alcoforado[1], **Agnieszka I. Bergel**[2], **Rui M.R. Cardoso**[3], **Alfredo D. Egídio dos Reis**[2], **Eugenio V. Rodriguez-Martinez**[4]

22 Aug
PS2

[1]*ISEG & CEMAPRE, Universidade de Lisboa & Universidade Federal de Pernambuco, alcoforado.renata@gmail.com*
[2]*ISEG & CEMAPRE, Universidade de Lisboa, agnieszka@iseg.ulisboa.pt, alfredo@iseg.ulisboa.pt*
[3]*Center for Mathematics and Applications (CMA), FCT NOVA and Department of Mathematics, FCT NOVA, rrc@fct.unl.pt*
[4]*BNP Paribas & CEMAPRE, evrodriguez@gmail.com*

Keywords: Ruin theory, dual risk model, penalty function, randomised observations, Erlang distribution

We consider the dual risk model with financial application, where the random gains (or claims, using the primal insurance risk model) occur under a renewal process. Due to the mathematical robustness of the model we do not need to impose the usual economic condition, as shown by [2], despite the fact that the ruin probability being one when the condition is not respected. We introduce a Gerber-Shiu type of penalty function applied to the dual model and consider that randomised observations are set in place as [1] do for the primal or classical compound Poisson insurance risk model.

We go further than these authors by studying a renewal risk process, mention in particular the Erlang(n) renewal model, and an independent observational Poisson process. Under these model ruin can only arise if it is indeed observed, we mean, the risk process may cross downwards the zero but ruin may not happen if it recovers before it can be observed.

We develop integral and differential equations, from which we study solutions for some cases, and particularly solve numerically for some examples in full. We show some figures and graphs from some chosen numerical examples. We compare them, where possible, with those of [1] although worked for the primal compound Poisson risk model. Also with those of [3].

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References

[1] Albrecher, H., Cheung, E. C., and Thonhauser, S. (2013). Randomized observation periods for the compound poisson risk model: the discounted penalty function. *Scandinavian Actuarial Journal* 2013(6), 424–452.

[2] Alcoforado, R. G., Bergel, A. I., Cardoso, R. M. R., Egídio dos Reis, A. D., and Rodríguez-Martínez, E. V. (2021). Ruin and dividend measures in the renewal dual risk model. *Methodology and Computing in Applied Probability*, https://doi.org/10.1007/s11009-021-09876-4

[3] Rodríguez-Martínez, E. V., Cardoso, R. M. R., and Egídio dos Reis, A. D. (2015). Some advances on the Erlang(n) dual risk model. *ASTIN Bulletin* 45(1), 127–150.

Social welfare under restricted risk classification

Indradeb Chatterjee[1], **Angus S. Macdonald**[2], **Pradip Tapadar**[1] **and R. Guy Thomas**[1]

[1]*School of Mathematics, Statistics and Acturial Science, University of Kent, UK*
[2]*Department of Actuarial Mathematics and Statistics, and the Maxwell Institute for Mathematical Sciences, Heriot-Watt University, UK*

23 Aug
PS5

Keywords: social welfare, relative utilitarianism, insurance risk classification, demand elasticity

This paper considers the effect of bans on insurance risk classification on utilitarian social welfare. We consider two regimes: full risk classification, where insurers charge the actuarially fair premium for each risk, and pooling, where risk classification is banned and for institutional or regulatory reasons, insurers do not attempt to separate risk classes, but charge a common premium for all risks.

For iso-elastic insurance demand, we derive sufficient conditions on higher and lower risks' demand elasticities which ensure that utilitarian social welfare is higher under pooling than under full risk classification. Using the concept of arc elasticity of demand, we extend the results to a form applicable to more general demand functions. Empirical evidence suggests that the required elasticity conditions for social welfare to be increased by a ban may be realistic for some insurance markets.

Authors have already published the results in [1].

References

[1] Chatterjee, I., Macdonald, A.S., Tapadar, P. and Thomas, R.G.(2021). When is utilitarian welfare higher under insurance risk pooling ? *Insurance: Mathematics and Economics* 101(B), 289–301.

How mortality and/or longevity risks impact log-optimal portfolio?

23 Aug
PS5

Tahir Choulli[1] **and Sina Yansori**[2]

[1]*University of Alberta, Edmonton, Alberta, tchoulli@ualberta.ca*
[2]*CIBC Risk Analytics and Forecasting, Toronto, Canada*

Keywords: death time, Log-optimal portfolio, numéraire portfolio, deflators, informational risks, utility, progressive enlargement of filtration, asymmetries of information, semimartingales and predictable characteristics

We consider a market model with two flows of information, which are mathematically represented by two filtrations. The smaller flow $\mathbb{F}$ models the information about the financial market which is available to all agents. The larger flow $\mathbb{G}$ incorporates additional information about the occurrence of a death time τ. As death times can not be seen before their occurrence, this boils down to suppose mathematically that $\mathbb{G}$ is the progressive enlargement of $\mathbb{F}$ with τ. In this setting, we quantify the impact of τ on the log-optimal portfolio when passing from the model $(S, \mathbb{F}, P)$ to the model $(S^\tau, \mathbb{G}, P)$. In particular, we will answer the following:

1. Which types of risks induced by τ that really affect the Log-portfolio portfolio?
2. What are the conditions on τ (preferably in terms of information theoretic concepts such as entropy) that guarantee the existence of the log-optimal portfolio of $(S^\tau, \mathbb{G}, P)$ when that of $(S, \mathbb{F}, P)$ already exists?
3. How Log-optimal portfolio can be described using the parameters of τ and those of the initial model $(S, \mathbb{F}, P)$?
4. What are the factors that fully determine *the increment in maximum expected logarithmic utility from terminal wealth* for the models $(S^\tau, \mathbb{G}, P)$ and $(S, \mathbb{F}, P)$, and how to quantify them?
5. Comparing between our progressive setting and the insider setting in which the additional information is known from the beginning.

This talk is based on works [1, 2, 3] with Sina Yansori (a former PhD).

References

[1] Choulli, T., Yansori, S. (2022). Log-optimal and numéraire portfolios for market models stopped at a random time. To appear in *Finance and Stochastics* .

[2] Choulli, T., Yansori, S. (2021): Explicit description of all deflators for markets under random horizon with application to NFLVR. Preprint available at arXiv:1803.10128v4 (2021).

[3] Choulli, T., Yansori, S: Log-optimal portfolio without NFLVR: Existence, complete characterization and duality. To appear in Theory of Probability and its Applications, Vol. 67, Issue 2. Available at arXiv:1807.06449v1 (2018).

A multi-curve HJM factor model for pricing and risk management

Tobias Bienek[1], **Griselda Deelstra**[2], **Andreas Lichtenstern**[1] **and Rudi Zagst**[1]

23 Aug
PS4

[1]*Technical University of Munich, Germany, Zagst@tum.de*
[2]*Université libre de Bruxelles, Belgium, Griselda.Deelstra@ulb.be*

Keywords: XIBOR rate, multiple yield curves, multiplicative spread, affine processes

The crisis that hit the financial markets in 2008 had an enormeous influence upon interest-rate modelling. Since August 2007, spreads between interbank offered rates, like e.g. LIBOR and EURIBOR rates, and overnight-indexed swap (OIS) rates have been significant. Historical spreads are (mostly) positive, are correlated and are ordered in function of the tenors.

In this paper, we will present a multiple-curve model which can be used both for risk management and for pricing since we will model it under both the real-world measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$. Indeed, in this paper, we first consider a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, \mathbb{P})$ where $\mathbb{P}$ is the historical probability, also called the physical or real-world measure. We concentrate upon a model with multiplicative relative spreads, inspired by the HJM and affine factor approaches, which allows for correlated, positive and ordered spreads, as well as tractable valuation formulas for some derivatives with optionality features. We will introduce the notion of δ_i-*positive multi-curve models* which summarizes the properties of positive multiplicative spreads and ordered multiplicative relative spreads. We start by formulating an HJM model under $\mathbb{P}$ and $\mathbb{Q}$ for both the OIS and XIBOR rates. For the latter, we will focus upon δ_i-XIBOR relative (instantaneous) forward rates $f_{i/i-1}(t,T)$, in order to capture more easily empirical observations with respect to different tenors. We derive appropriate XIBOR HJM drift constraints, describe the dynamics of the different forward rates and spreads under different measure changes (including forward measures) and propose equivalent properties to check the δ_i-positivity. We further specify an explicit δ_i-positive model (under $\mathbb{P}$ and $\mathbb{Q}$) based upon affine spread factor processes, which satisfies the XIBOR HJM drift constraint as well as the δ_i-positivity constraints. Therefore, we use a Hull-White model for the OIS short rates since the OIS rate can turn negative. On the other hand, we base the δ_i-XIBOR relative (instantaneous) forward rates $f_{i/i-1}(t,T)$ upon Cox-Ingersoll-Ross (CIR) processes. In this explicit model, we show that it is easy to price linear financial products as well as interest-rate derivatives with optionality features. As an example, we derive the price of a caplet and of options with a payoff based upon XIBOR forward prices with different tenors.

We perform on one hand a calibration of the model based upon cap prices. On the other hand, we do a full estimation of our proposed model under the historical probability by using a Kalman filter approach. Numerical results are included, and they confirm that the model performs very well.

The use of autoencoders for training deep neural networks with mixed categorical and numerical features

24 Aug
PS7

Łukasz Delong[1] **and Anna Kozak**[2]

[1] *SGH Warsaw School of Economics, Institute of Econometrics, lukasz.delong@sgh.waw.pl*
[2] *R&D Department Quantee, anna.kozak@quantee.ai*

Keywords: autoencoders, corruption of inputs, categorical and numerical features, embeddings, representation learning

We focus on modelling categorical features and improving predictive power of deep neural networks with mixed categorical and numerical features. First, we study regular and denoising autoencoders for categorical features in unsupervised learning problems. Second, we discuss possible architectures of neural networks in supervised learning problems which differ in the way categorical features are concatenated with numerical features. Third, we investigate a learning algorithm where we initialize parameters of a neural network in subsequent layers with representations of inputs learned with autoencoders for categorical and numerical data. We illustrate our techniques on a real insurance data set with motor claim numbers. We show that our new architecture of a neural network initialized with parameters derived from autoencoders and a joint embedding for all categorical features performs better, in terms of predictive power and stability of calibrations, than the classical architecture with random initialization of parameters and separate entity embeddings for each categorical feature. The approach from this paper could serve as an alternative modelling technique used for training deep neural networks in practice.

Risk aggregation under IFRS 17: An ultimate run-off adaptation of Solvency 2 elliptic aggregation

23 Aug
PS4

Tachfine El Alami[1,2], **Laurent Devineau and Stéphane Loisel**[1]

[1]*ISFA, Univ Lyon 1, UCBL, LSAF EA2429, F-69007, Lyon, France*
[2]*ADDACTIS France, Paris, France*

Keywords: IFRS 17, Solvency 2, risk adjustment, risk aggregation, correlation, time diversification, ultimate run-off

The standard IFRS 17 requires a risk adjustment to reflect the compensation the insurance entity requires for bearing the uncertainty associated with non-financial risks. The risk adjustment is one of the primary calculations in IFRS 17 disclosures and is an influential factor to profit reporting and allocation over time. IFRS 17 does not prescribe any specific techniques on calculation methodologies; insurance entities are free to adopt their own while meeting several qualitative rules to ensure its consistency. This paper focuses on the recommendations of the paragraph §B88 stating that the risk adjustment is required to reflect the degree of diversification benefit for bearing the risk. We suggest a method for aggregating elementary risk adjustments based on the Solvency 2 elliptical aggregation. We introduce the concept of ultimate run-off correlation as opposed to Solvency's one-year correlation and provide a theoretical bridge between both depending on a time diversification parameter. We explore correlation structures involving this time diversification and discuss analytical properties in terms of possible correlations values and the resulting impact on the aggregated RA behavior.

References

[1] IASB IFRS 17 Insurance Contracts incorporating the June 2020 amendments (2020). International Accounting Standards Board.

[2] IASB Basis for Conclusions - IFRS 17 Insurance Contracts (2017). International Accounting Standards Board.

[3] IASB Illustrative Examples - IFRS 17 Insurance Contracts (2017). International Accounting Standards Board.

[4] Devineau, L. and Loisel, S. (2009). Risk aggregation in Solvency II: How to converge the approaches of the internal models and those of the standard formula?

[5] Bernard, C., Jiang, X. and Wang, R. (2014). Risk aggregation with dependence uncertainty. *Insurance: Mathematics and Economics* 54, 93-108.

[6] Embrechts, P. and Puccetti, G. (2010). Risk aggregation. *Copula Theory and Its Applications*, pp. 111-126, Springer.

[7] Filipović, D. (2009). Multi-level risk aggregation. *ASTIN Bulletin: The Journal Of The IAA* 39, 565-575.

[8] Rosenberg, J. and Schuermann, T. (2006). A general approach to integrated risk management with skewed, fat-tailed risks. *Journal Of Financial Economics* 79, 569-614.

[9] Lee, G. and Shi, P. (2019). A dependent frequency–severity approach to modeling longitudinal insurance claims. *Insurance: Mathematics and Economics* 87, 115-129.

[10] Shi, P., Feng, X. and Boucher, J. (2016). Multilevel modeling of insurance claims using copulas. *The Annals of Applied Statistics* 10, 834-863.

[11] Czado, C., Schepsmeier, U. and Min, A. (2012). Maximum likelihood estimation of mixed C-vines with application to exchange rates. *Statistical Modelling* 12, 229-255.

[12] Denuit, M., Dhaene, J., Goovaerts, M., Kaas, R. (2006). *Actuarial theory for dependent risks: measures, orders and models*. John Wiley & Sons.

[13] Cherubini, U., Luciano, E., Vecchiato, W. (2004). *Copula methods in finance*. John Wiley & Sons.

[14] McNeil, A., Frey, R. and Embrechts, P. (2015). *Quantitative risk management: concepts, techniques and tools-revised edition*. Princeton university press.

[15] CEIOPS QIS 4 Technical Specifications (2008). Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS)

[16] Kotz, S. and Nadarajah, S. (2004). *Multivariate t-distributions and their applications*. Cambridge University Press.

[17] Ding, P. (2016). On the conditional distribution of the multivariate t distribution. *The American Statistician* 70, 293-295.

[18] Böttcher, A. and Grudsky, S. (2005). *Spectral properties of banded Toeplitz matrices*. SIAM.

[19] Dedecker, J., Doukhan, P., Lang, G., Rafael, L., Louhichi, S., Prieur, C. (2007). Weak dependence. *Weak Dependence: With Examples And Applications*, pp. 9-20, Springer.

[20] Makhoul, J. (1981). On the eigenvectors of symmetric Toeplitz matrices. *IEEE Transactions On Acoustics, Speech, And Signal Processing* 29, 868-872.

Convergence and robustness to model uncertainty in multiple period valuation problems

23 Aug
PS6

Nils Engler[1] **and Filip Lindskog**[2]

[1]*Stockholm University, Sweden, nils.engler@math.su.se*
[2]*Stockholm University, Sweden, lindskog@math.su.se*

Keywords: valuation, multiple period models, conditional weak convergence

Valuation and risk measurement in multiple period models, see e.g. [1, 2, 3, 4], naturally lead to values that can be expressed as compositions of mappings which are conditional risk measures or conditional monetary utility functions. Computing such values explicitly by backward induction is rarely possible and approximation using methods such as Least Squares Monte Carlo is often necessary.

In this paper we analyse how changes in the multiple period model affects multiple period values. More precisely, we present results where conditional weak convergence for the multiple period model imply convergence of multiple period values expressed as compositions of conditional risk measures and conditional monetary utility functions. We focus in particular on conditional weak convergence to Gaussian limits which enables explicit computation of the corresponding limits of compositions. A particular class of cash flow models appearing naturally in non-life insurance modelling is studied in detail.

References

[1] Engsner, H., Lindholm, M., Lindskog, F. (2017). Insurance valuation: A computable multi-period cost-of-capital approach. *Insurance Mathematics and Economics* 72, 250–264.

[2] Föllmer, H., Schied, A. (2016). *Stochastic Finance: An Introduction in Discrete Time*. De Gruyter, Berlin, Boston.

[3] Möhr, C. (2011). Market-Consistent Valuation of Insurance Liabilities by Cost of Capital. *ASTIN Bulletin* 41(2), 315–341.

[4] Pelsser, A., Stadje, M. (2014). Time-Consistent and Market-Consistent Evaluations. *Mathematical Finance* 24(1), 25–65.

Ruin and profitability in Bitcoin mining: analysis of pools and empirical evidence

22 Aug
PS2

Hansjörg Albrecher[1,2], **Dina Finger**[1] **and Pierre-O. Goffard**[3]

[1]*University of Lausanne, Switzerland*
[2]*Swiss Finance Institute*
[3]*Université de Lyon 1, France*

Keywords: ruin theory, risk sharing, blockchain, Bitcoin mining

The resource-consuming mining of blocks on a blockchain equipped with a proof of work consensus protocol bears the risk of ruin, namely when the operational costs for the mining exceed the received rewards. In this talk we will discuss to what extent it is of interest to join a mining pool that reduces the variance of the return of a miner for a specified cost for participation. Using insights and techniques from ruin theory and risk sharing in insurance, we quantitatively study the effects of pooling in this context and derive several explicit formulas for quantities of interest. The results will be illustrated in numerical examples for parameters of practical relevance. Furthermore, some empirical evidence from the Bitcoin market will be presented and compared to the theoretical results. In particular, we consider transaction fees attached to a block as part of the miner's income.

On a risk process with deterministic investment and multiplicative jumps – an application to poverty trapping

22 Aug
PS2

Kira Henshaw[1], José Miguel Flores-Contró[2], Sooie-Hoe Loke[3], Corina Constantinescu[4], Séverine Arnold[5] and Jorge Mario Rámirez Osorio[6]

[1]*University of Liverpool, United Kingdom, k.henshaw@liverpool.ac.uk*
[2]*University of Lausanne, Switzerland, josemiguel.florescontro@unil.ch*
[3]*Central Washington University, United States of America, sooiehoe.loke@cwu.edu*
[4]*University of Liverpool, United Kingdom, c.constantinescu@liverpool.ac.uk*
[5]*University of Lausanne, Switzerland, severine.arnold@unil.ch*
[6]*Universidad Nacional de Colombia, Colombia, jmramirezo@unal.edu.co*

Keywords: household capital, poverty traps, trapping probability, risk process, Gerber-Shiu function

In this article, we consider a risk process with a deterministic investment and multiplicative jump (collapse) structure to model the capital of a household. Focusing on the trapping time of such a process, where trapping occurs when the capital level of a household falls into the area of poverty, from which it is difficult to escape without external help, we introduce a function analogous to the classical Gerber-Shiu expected discounted penalty function, which incorporates information on the trapping time, the capital surplus immediately before trapping and the capital deficit at trapping. The trapping time is analysed in terms of its Laplace transform via the infinitesimal generator of the capital process. Considering the remaining proportion of capital to be distributed as a special case of the beta distribution, closed-form expressions for quantities typically studied in classical risk theory, including the capital deficit at trapping and the trapping probability, are obtained.

The role of direct cash transfers towards extreme poverty alleviation – an Omega risk process

22 Aug
PS2

José Miguel Flores-Contró[1] **and Séverine Arnold**[2]

[1]*University of Lausanne, Switzerland, josemiguel.florescontro@unil.ch*
[2]*University of Lausanne, Switzerland, severine.arnold@unil.ch*

Keywords: poverty traps, probability of extreme poverty, Omega risk process, cash transfers, cost of social protection

Trapping refers to the event when a household falls into the area of poverty. Households that live or fall into the area of poverty are said to be in a poverty trap, where a poverty trap is a state of poverty from which it is difficult to escape without external help. Similarly, extreme poverty is considered as the most severe type of poverty, in which households experience severe deprivation of basic human needs. In this article, we consider an Omega risk process with deterministic growth and a multiplicative jump (collapse) structure to model the capital of a household. It is assumed that, when a household is not trapped, its capital surplus grows exponentially, whereas once trapped, the capital grows linearly only due to the external support of direct transfers (cash transfers) of money provided by donors or governments. Under this model, we derive closed-form expressions of the probability of extreme poverty, which only depends on the value of the capital at that time given by some extreme poverty rate function. For different extreme poverty rate functions, we present numerical examples to illustrate the role of cash transfers on extreme poverty dynamics.

Optimal multiperiod mixture between pay-as-you-go and funded financial systems for social security

23 Aug
PS4

Ivan Alexis Fonseca Diaz[1] **and Séverine Arnold**[2]

[1]*University of Lausanne, Switzerland, IvanAlexis.FonsecaDiaz@unil.ch*
[2]*University of Lausanne, Switzerland, Severine.Arnold@unil.ch*

Keywords: social security, Nash equilibrium, Pareto optimality, time-consistency, pay-as-you-go, funded financial systems

As an instrument to transfer part of the income of individuals in productive ages (active period) to their future unproductive lifetime (retirement period), social security systems could be assessed according to their ability to maximize the accumulated wealth of working generations at the time of their retirement. In the existing literature, the answer to the question of whether fully funded systems are superior or not to pay-as-you-go systems remains unclear, as both financial systems deal with different advantages and limitations in terms of risk and profitability in the accumulation phase. Recently, mixtures of both systems have been proposed. However, given the impossibility to reinvest the capital allocated in the pay-as-you-go part, the system remains time-inconsistent in the multiperiod case. In our research, we present a Nash equilibrium and Pareto optimal capital allocation of coexisting generations investing between pay-as-you-go and fully funded systems. In addition to reaching the cooperative and non-cooperative optimal welfare at every time, our model guarantees time consistency in the multiperiod case. Our framework also allows us to show that optimality is not reached by using a pure pay-as-you-go or a pure fully funded system, but by a mixture of both systems. Furthermore, as the optimality of this investment strategy is measured under a Mean Variance criterion, this model also represents an alternative solution to the well-known time inconsistency issue of investment strategies derived under this criterion.

Extension of as-if-Markov modeling to scaled payments

Marcus C. Christiansen[1] **and Christian Furrer**[2]

[1]*University of Oldenburg, Germany*
[2]*University of Copenhagen, Denmark, furrer@math.ku.dk*

22 Aug
PS3

Keywords: life insurance, non-Markov models, Kolmogorov's forward equations, incidental policyholder behavior, landmark estimators

In multi-state life insurance, as-if-Markov modeling has recently been suggested as an alternative to Markov modeling in case of deterministic sojourn and transition payments. Incidental policyholder behavior, on the other hand, gives rise to duration-dependent payments in the form of so-called scaled payments. This talk establishes as-if-Markov modeling also for scaled payments. To this end, we employ change of measure techniques to transfer the added complexity from the payments to an auxiliary probabilistic model. Based hereon, we show how to compute the accumulated cash flow by solving a system of equations comparable to Kolmogorov's forward equations for Markov chains, but with the transition rates replaced by certain forward transition rates related to the auxiliary probabilistic model. Finally, we provide feasible landmark estimators for these auxiliary forward transition rates subject to right-censoring.

On the use of convolution order for expected utility differentiation and optimization

23 Aug
PS6

Romain Gauchon[1] **and Karim Barigou**[2]

[1]*ISFA, Univ Lyon 1, UCBL, LSAF EA2429, France; Pericles actuarial rgauchon@pericles-actuarial.com*
[2]*ISFA, Univ Lyon 1, UCBL, LSAF EA2429, France, karim.barigou@univ-lyon1.fr*

Keywords: convolution order, expected utility, fractional calculus

Deriving a parametric random variable with respect to one of its parameters is usually considered as a difficult task. Indeed, due to the randomness, the limit that we call derivative is not well defined. However, ordering random variables using the convolution order can help to tackle such problem. This order naturally involves a small increase of a random variable, and allows us to investigate some quantities such as a random variable derivative. Ultimately, if we consider some classical utility functions and a family of parametric random variables ordered with the convolution order, we manage to obtain some formulas for the expected utility derivative. The economic interest of such formulas, as well as an economic example, will be provided.

Risk-sharing in equity-linked insurance products: Stackelberg equilibrium between an insurer and a reinsurer

23 Aug
PS6

Yevhen Havrylenko[1], **Maria Hinken**[2] **and Rudi Zagst**[1]

[1]*Technical University of Munich, Germany, yevhen.havrylenko@tum.de, zagst@tum.de*
[2]*University of Ulm, Germany, maria.hinken@uni-ulm.de*

Keywords: portfolio optimization, life insurance, reinsurance, Stackelberg game

Common investment strategies ensuring guaranteed returns in equity-linked insurance products are challenged nowadays by low interest rates [2, 3]. Thus, we study an alternative strategy when an insurance company shares financial risk with a reinsurance company. We model this situation as a Stackelberg game, which is a bi-level optimization problem. As the number of reinsurance companies is significantly lower than the number of primary insurance companies and reinsurance companies have a rather dominant position [1], the reinsurer is the leader in the Stackelberg game that we consider. The reinsurance company maximizes its expected utility by selecting its optimal investment strategy and a safety loading in the reinsurance contract it offers to the insurer. The reinsurer can assess how the insurer will rationally react on each action of the reinsurer. The insurance company is the follower and maximizes its expected utility by choosing its investment strategy and the amount of reinsurance the company purchases at the price offered by the reinsurer. In this game, we derive the Stackelberg equilibrium for general utility functions. For power utility functions, we calculate the equilibrium explicitly and find that the reinsurer selects the largest reinsurance premium such that the insurer may still buy the maximal amount of reinsurance. Since in the equilibrium the insurer is indifferent in the amount of reinsurance, in practice, the reinsurer should consider charging a smaller reinsurance premium than the equilibrium one. Therefore, we propose several criteria for choosing such a discount rate and investigate its wealth-equivalent impact on the utilities of both parties.

References

[1] Chen, L., and Shen, Y. (2018). On a new paradigm of optimal reinsurance: A stochastic Stackelberg differential game between an insurer and a reinsurer. *Journal of Economic Dynamics and Control* 48(2), 905–960.

[2] Escobar-Anel, M., Havrylenko, Y., Kschonnek, M., and Zagst, R. (2022). Decrease of capital guarantees in life insurance products: can reinsurance stop it? *Insurance: Mathematics and Economics* 105, 14–40.

[3] Havrylenko, Y., Hinken, M., and Zagst, R. (2022). Risk sharing in equity-linked insurance products: Stackelberg equilibrium between an insurer and a reinsurer. Pre-print available at https://arxiv.org/pdf/2203.04053.pdf.

A probability of ruin approach to optimize pension fund investments

23 Aug
PS4

Abraham Hernández-Pacheco[1], **Agnieszka I. Bergel**[2], **and Alfredo D. Egídio dos Reis**[3]

[1]*ISEG & CEMAPRE, Universidade de Lisboa, abe@vitalis.com.mx*
[2]*ISEG & CEMAPRE, Universidade de Lisboa, agnieszka@iseg.ulisboa.pt*
[3]*ISEG & CEMAPRE, Universidade de Lisboa, alfredo@iseg.ulisboa.pt*

Keywords: defined benefit pensions, ruin theory, minimum ruin portfolio

We use a novel concept of ruin probabilities to optimize the asset allocation for various asset classes in a pension fund. We look at a sponsor of a defined benefit plan, where current assets plus the expected present value of future contributions are smaller than the expected present value of its liabilities discounted at a "risk free" interest rate; such is the case of thousands of pension plans worth trillions of $USD. This proposal comes an alternative to the Solvency II framework.

We present an alternative methodology to the classical Asset Liability Management techniques that consider the long-term effects of returns *versus* volatility, along with funding levels and funding policy. For several combinations of asset allocation, with a proper concept of ruin probability, newly defined, our approach estimates the portfolio's probability of ruin. We particularly study the asset allocation of a portfolio that minimizes the probability of reaching a threshold, which either defines the need to increase contributions or curtail benefits (our ruin concept), under a set of various assumptions of initial funding, future contributions, and financial forecasting models. We will call such portfolio the Minimum Ruin Probability (MRP) portfolio.

Our study also comes as sequel and development of the paper by Hernández-Pacheco and Salgado [1]. We explain the idea and rationale of using the probability of ruin to optimize asset allocation, its relation with other optimization techniques and novel empirical preliminary results.

References

[1] Hernández-Pacheco, A., Salgado, J. (2015). Optimal asset allocation for defined benefit plans under a heavy-tailed-coupled portfolio. In *International Actuarial Colloquium, IAA PBSS Life/IACA Colloquium, Oslo* IAA.

Mortality credits within large survivor funds

Michel Denuit[1], **Peter Hieber**[2] **and Christian Y. Robert**[3]

22 Aug
PS1

[1]*UC Louvain, Belgium, michel.denuit@uclouvain.be*
[2]*Université de Lausanne (HEC Lausanne), Switzerland, peter.hieber@unil.ch*
[3]*ENSAE Paris, France, chrobert@ensae.fr*

Keywords: mortality risk pooling, tontine, conditional mean risk sharing

Survivor funds are financial arrangements where participants agree to share the proceeds of a collective investment pool in a pre-described way depending on their survival. This offers investors a way to benefit from mortality credits, boosting financial returns. Following [1], participants are assumed to adopt the conditional mean risk sharing rule introduced in [2] to assess their respective shares in mortality credits. This talk looks at pools of individuals that are heterogeneous in terms of their survival probability and their contributions. Imposing mild conditions, we show that individual risk can be fully diversified if the size of the group tends to infinity. For large groups, we derive simple, hierarchical approximations of the conditional mean risk sharing rule.

References

[1] Denuit, M. (2019). Size-biased transform and conditional mean risk sharing, with application to P2P insurance and tontines. *ASTIN Bulletin* 49, 591-617.

[2] Denuit, M., Dhaene, J. (2012). Convex order and comonotonic conditional mean risk sharing. *Insurance: Mathematics and Economics* 51, 265-270.

[3] Denuit, M., Hieber, P., Robert, C. Y. (2022). Mortality credits within large survivor funds, *working paper*.

[4] Hieber, P., Lucas, N. (2022). Modern life-care tontines. *ASTIN Bulletin*, in press.

Neural networks for frequency-severity modelling: a benchmark study from data preprocessing steps to technical tariff

24 Aug
PS7

Freek Holvoet[1], **Katrien Antonio**[1,2] **and Roel Henckaerts**[1,3]

[1]*KU Leuven, Belgium, freek.holvoet@kuleuven.be; katrien.antonio@kuleuven.be*

[2]*University of Amsterdam, The Netherlands*

[3]*Prophecy Labs, 1000 Brussels, Belgium*

Keywords: frequency-severity modelling, neural networks, interpretability, machine learning, data science

Insurers usually turn to generalized linear models for modelling claim frequency and severity data. Due to their success in other fields, machine learning techniques are gaining more popularity within the actuarial toolbox. This paper investigates the use of artificial neural networks (ANN) as well as the recently proposed combined actuarial neural networks (CANN, see [4]) for both claim frequency and severity modelling. Our data preprocessing steps put focus on the multiple types of input features typically present in insurance pricing use cases. We use autoencoders to embed the categorical variables into the neural networks [1] and focus on their multi-task advantages in a frequency-severity setting. Interpretation techniques provide insight into both the claim frequency and the severity models, as well as the resulting technical tariff structure [2]. A benchmark study with three insurance data sets allows to evaluate the robustness of our findings. Finally, we use the workflow proposed in [3] to construct global surrogate models for the neural nets' frequency-severity models. These surrogates enable the translation of the essential insights captured by the ANNs and/or CANNs to generalized linear models (GLMs), within the actuarial comfort zone.

References

[1] Delong, L. and Kozak, A. (2021). The use of autoencoders for training neural networks with mixed categorical and numerical features. *SSRN Electronic Journal*, `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3952470`.

[2] Henckaerts, R., Côté, M.-P., Antonio, K. and Verbelen, R. (2021). Boosting Insights in Insurance Tariff Plans with Tree-Based Machine Learning Methods. *North American Actuarial Journal*.

[3] Henckaerts, R., Antonio, K. and Côté, M.-P. (2022). When stakes are high: balancing accuracy and transparency with Model-Agnostic Interpretable Data-driven suRRogates. *Expert Systems with Applications*.

[4] Wüthrich, M. and Merz, M. (2018). Editorial: Yes we CANN!. *ASTIN Bulletin*.

Efficient numerical computation of the ruin probability

22 Aug
PS1

Naoyuki Ishimura[1], **Hiroko Soutome**[2] **and Hitoshi Imai**[3]

[1]*Chuo University, Japan, naoyuki@tamacc.chuo-u.ac.jp*
[2]*Teikyo University, Japan, hsoutome@main.teikyo-u.ac.jp*
[3]*Doshisha University, Japan, himai@mail.doshisha.ac.jp*

Keywords: ruin probability, integral equation, numerical scheme

We are concerned with the numerical computation of the next Volterra type integral equation of the second kind:

$$1-\phi(u)=\frac{\lambda}{c}\int_u^\infty (1-F_X(y))dy+\frac{\lambda}{c}\int_0^u (1-F_X(u-y))(1-\phi(y))dy.$$

As is well known, this type of integral equation arises in many scientific situation. The present equation, which is (11.3.5) in [2] (see also [1]), describes the probability of ultimate ruin $1-\phi(u)$, where $\phi(u)$ means the probability of ultimate survival under the classical Cramer-Lundberg model. Here λ denotes the parameter of a Poisson process which models the number of claim process, and $c\ (>0)$ is the insurer's constant rate of premium income per unit time. $F_X(x)$ denotes the distribution function of i.i.d. random variables for individual claim amounts.

Our aim is to provide efficient numerical scheme for computing $\phi(u)$. Our procedure is based on the Chebyshev-Gauss-Radau collocation method combined with the double exponential formula for the computation of integral. First, in order to treat the infinite interval properly, we transform the interval $\{u>0\}$ into a bounded interval $\{-1<x\le 1\}$ by $u(x)=2(1-x)/(1+x)(3-x)$. We put $\overline{\phi}(x):=\phi(u(x))$ and define the approximate solution:

$$\overline{\phi_N}(x)=\sum_{j=0}^{N} s_j(x)\overline{\phi}(x_j),\qquad \text{where}\quad x_j=\cos\frac{2j\pi}{2N+1}\quad (0\le j\le N),$$

$$s_j(x)=\frac{4}{(2N+1)\overline{c}_j}\sum_{k=0}^{N}\frac{1}{\overline{c}_k}T_k(x_j)T_k(x),\qquad \overline{c}_n=\begin{cases}2 & (n=0)\\ 1 & (n=1,\cdots,N)\end{cases},$$

and $T_k(x)=\cos(k\cos^{-1}x)$. By the initial condition, we have $\overline{\phi}(x_0)=1-\frac{\lambda}{c}E[X]$. We then compute $\{\overline{\phi}(x_j)\}$ from the discretized system of equations. In particular, we are able to numerically compute $\phi(\infty)$. Numerical experiments show that our scheme is efficient and accurate.

References

[1] Dickson, D.C.M. (2017). *Insurance Risk and Ruin, 2nd Edition*. Cambridge University Press, Cambridge.

[2] Panjer, H.H., Wilmot, G.E. (1992). *Insurance Risk Models*. Society of Actuaries, Schaumburg, IL.

A general surplus decomposition principle in life insurance

22 Aug
PS2

Julian Jetses

Carl von Ossietzky Universität Oldenburg, Germany, julian.jetses@uni-oldenburg.de

Keywords: with-profit life insurance, profit and loss attribution, bonus and dividends, sequential decompositions

In with-profit life insurance, the prudent valuation of future insurance liabilities leads to systematic surplus that mainly belongs to the policyholders and is redistributed as bonus. For a fair and lawful redistribution of surplus the insurer needs to decompose the total portfolio surplus with respect to the contributions of individual policies and with respect to different risk sources. For this task, actuaries have a number of heuristic decomposition formulas, but an overarching decomposition principle is still missing. Therefore, we introduce the so-called ISU decomposition principle that bases on infinitesimal sequential updates of the insurer's valuation basis, and picture its link to the existing heuristic decomposition formulas. Furthermore, alternative decomposition principles and their relation to the ISU decomposition principle are discussed.

References

[1] Jetses, J., Christiansen, M. C. (2022). A General Surplus Decomposition Principle in Life Insurance. *Scandinavian Actuarial Journal*, URL: https://doi.org/10.1080/03461238.2022.2049636.

Representation of concave distortions and applications

Gero Junike

Carl von Ossietzky University, gero.junike@uol.de

22 Aug
PS3

Keywords: representation of distortion functions, premium principle, coherent risk measure, WANG-transform, log-concavity

A family of concave distortion functions is a set of concave and increasing functions, mapping the unity interval onto itself. Distortion functions play an important role defining coherent risk measures. We prove that any family of distortion functions which fulfils a certain translation equation, can be represented by a distribution function. An application can be found in actuarial science: moment-based premium principles are easy to understand but in general are not monotone and cannot be used to compare the riskiness of different insurance contracts with each other. Our representation theorem makes it possible to compare two insurance risks with each other consistent with a moment-based premium principle by defining an appropriate coherent risk measure.

Dual formulation of the optimal consumption problem with multiplicative habit formation

23 Aug
PS6

Thijs Kamma[1] **and Antoon Pelsser**[2]

[1]*Maastricht University & Netspar, The Netherlands, t.kamma@maastrichtuniversity.nl*
[2]*Maastricht University, University of Amsterdam & Netspar, The Netherlands, a.pelsser@maastrichtuniversity.nl*

Keywords: Fenchel duality, habit formation, life-cycle investment, stochastic optimal control, utility maximisation

This paper provides a dual formulation of the optimal consumption problem with internal multiplicative habit formation. In this problem, the agent derives utility from the ratio of consumption to the internal habit component. Due to this multiplicative specification of the habit model, the optimal consumption problem is not strictly concave and incorporates irremovable path-dependency. As a consequence, standard Lagrangian techniques fail to supply a candidate for the corresponding dual formulation. Using Fenchel's Duality Theorem, we manage to identify a candidate formulation and prove that it satisfies strong duality. On the basis of this strong duality result, we develop an evaluation mechanism to measure the accuracy of analytical or numerical approximations to the optimal solutions.

Assessing model uncertainty for log-symmetric distributions

Carole Bernard[1], **Rodrigue Kazzi**[2] **and Steven Vanduffel**[3]

22 Aug
PS3

[1]*Grenoble Ecole de Management, France, and Vrije Universiteit Brussel, Belgium, carole.bernard@grenoble-em.com*
[2]*Vrije Universiteit Brussel, Belgium, rodrigue.kazzi@vub.be*
[3]*Vrije Universiteit Brussel, Belgium, steven.vanduffel@vub.be*

Keywords: Value-at-Risk, risk bounds, model uncertainty, log-symmetry, unimodality

In this talk, we present upper and lower bounds for the Value-at-Risk of log-symmetric risks under two sets of (trusted) assumptions. First, we consider the case in which the log-transformed random variable is known to have a unimodal and symmetric distribution with a known mean and a known maximum variance. The bounds, in this case, are sharp and significantly improve over those available in the literature for all probability levels lower than some extreme value. However, the practical issue of this case is that the upper bound increases dramatically at extreme probability levels.

Second, we consider the case in which the random variable at hand is known to be log-symmetric and has a unimodal distribution with a known median, a known maximum mean, and a known second moment. For high probability levels, we could derive an upper bound that is sharp and stable for extreme probability levels and consequently offer a significant improvement over the bound of the first case (and the findings in the literature) for high probability levels. Indeed, this upper bound can be extended to the cases where the first moment is unknown but finite and the second moment is unknown and possibly infinite.

Our findings are illustrated using real-world datasets: an automobile insurance claims dataset and a general liability claims dataset (heavy-tailed).

Mandatory pension saving and homeownership

Marcel Fischer[1], **Bjarne Astrup Jensen**[2], **Marlene Koch**[3]

23 Aug
PS4

[1]*Copenhagen Business School, Denmark and University of Konstanz, Germany, marcel.fischer@uni.kn*
[2]*Copenhagen Business School, Denmark, ba.fi@cbs.dk*
[3]*University of Konstanz, Germany, marlene.koch@uni.kn*

Keywords: retirement saving, homeownership, pension system design, loan-to-value ratio, housing market entry

We explore the implications of mandatory minimum contributions to tax-deferred retirement accounts over the life cycle. These contributions defer housing market entry and increase loan-to-value ratios. We propose a flexible retirement saving scheme that does not force individuals to build up savings in a tax-deferred retirement account and only requires them to save in either a taxable account, a tax-deferred retirement account, or through home equity if they are undersaving. This flexible retirement saving scheme largely alleviates the unintended side effects of mandatory minimum contributions and simultaneously ensures that individuals build up sufficient savings for retirement.

Combining multi-asset and intrinsic risk measures

Christian Laudagé[1], **Jörn Sass**[2] **and Jörg Wenzel**[3]

22 Aug
PS3

[1]*Johannes Kepler University Linz, Austria, christian.laudage@jku.at*
[2]*Technische Universität Kaiserslautern, Germany, sass@mathematik.uni-kl.de*
[3]*Fraunhofer Institute for Industrial Mathematics ITWM, Germany, joerg.wenzel@itwm.fraunhofer.de*

Keywords: intrinsic risk measure, multi-asset risk measure, multiple eligible assets, diversification, expected shortfall

The risk of a future payoff is commonly quantified by calculating the costs of a hedging portfolio such that the resulting position is acceptable, i.e., that it passes a capital adequacy test. A multi-asset risk measure describes the minimal external capital which has to be raised into multiple eligible assets to make a future position acceptable. Recently, the alternative methodology of intrinsic risk measures was introduced in the literature. These ask for the minimal proportion of the financial position which has to be reallocated to pass the capital adequacy test, i.e., only internal capital is used.

In this talk we combine these two concepts and call this new type of risk measure a multi-asset intrinsic risk measure. It allows to secure the financial position by external capital as well as reallocating parts of the portfolio as an internal rebooking. We investigate several properties to demonstrate similarities and differences to the two aforementioned classical types of risk measures. We find that diversification reduces the capital requirement only in special situations depending on the financial positions. With the help of Sion's minimax theorem we also prove a dual representation for multi-asset intrinsic risk measures. Finally, we determine capital requirements in a model motivated by the Solvency II methodology.

Bias regularisation, dispersion modelling, and auto-tariffication

Mathias Lindholm

Stockholm University, Sweden, lindholm@math.su.se

23 Aug
PS6

Keywords: bias regularisation, local bias, dispersion modelling, auto-tariffication

It is well known that GLMs are globally unbiased when using canonical link-functions, and if other link-functions are used, it is easy to adjust the mean model to become globally unbiased. Further, unless the model structure is sufficiently flexible one would expect local bias. Local bias in a pricing model may lead to adverse selection effects. In dispersion modelling, local bias from a poor mean model may lead to a dispersion model that explains bias rather than true dispersion.

Moreover, analysis of bias for e.g. GLMs and so-called double GLMs depend on that certain assumptions in terms of properties of exponential families are satisfied. This work is not assuming data from an exponential family. Instead we consider distribution-free approaches for local bias adjustment, aiming to minimise the squared-error loss of piecewise constant bias-adjusted predictors. This is done using two different binning strategies taking a given predictor as the starting point for the selection of bins where cross validation is used to determine the optimal binning.

The resulting bias-adjusted predictors are piecewise constant on the set of bins selected by the bias-adjustment procedure. In a pricing context, this can be thought of as “auto-tariffication” where the resulting locally unbiased tariff has an optimal number of unique tariff cells.

Based on both simulated data and real insurance data, using both simple GLMs and GBMs, the resulting tariffs produced in this way will have a rather small number of tariff cells while maintaining or improving the predictive performance compared to the initial predictor.

The presentation is based on joint work together with H. Boström, F. Lindskog, and J. Palmquist.

Multiple-prior valuation of cash flows subject to capital requirements

23 Aug
PS6

Filip Lindskog

Stockholm University, Sweden, lindskog@math.su.se

Keywords: valuation, capital requirements, limited liability, multiple periods, multiple priors

We study market-consistent valuation of liability cash flows motivated by current regulatory frameworks for the insurance industry. Building on the theory on multiple-prior optimal stopping we propose a valuation functional with sound economic properties that applies to any liability cash flow. Whereas a replicable cash flow is assigned the market value of the replicating portfolio, a cash flow that is not fully replicable is assigned a value which is the sum of the market value of a replicating portfolio and a positive margin. The margin is a direct consequence of considering a hypothetical transfer of the liability cash flow from an insurance company to an empty corporate entity set up with the sole purpose to manage the liability run-off, subject to repeated capital requirements, and considering the valuation of this entity from the owner's perspective taking model uncertainty into account. Aiming for applicability, we consider a detailed insurance application and explain how the optimisation problems over sets of probability measures can be cast as simpler optimisation problems over parameter sets corresponding to parameterised density processes appearing in applications.

This talk is based on joint work with Hampus Engsner and Julie Thøgersen [1].

References

[1] Engsner, H., Lindskog, F, Thøgersen, J. (2021). Multiple-prior valuation of cash flows subject to capital requirements. *arXiv:2109.00306*

Optimal prevention strategies in the classical risk model

Romain Gauchon[1], **Stéphane Loisel**[1], **Jean-Louis Rullière**[1] **and Julien Trufin**[2]

22 Aug
PS1

[1]*Univ. Lyon 1, France, romain.gauchon@laposte.net; loisel@univ-lyon1.fr; jean-louis.rulliere@univ-lyon1.fr*
[2]*Université Libre de Bruxelles, Belgium, julien.trufin@ulb.be*

Keywords: ruin theory, prevention

In this talk, we propose and study a first risk model in which the insurer may invest into a prevention plan which decreases claim intensity. We determine the optimal prevention investment for different risk indicators. In particular, we show that the prevention amount minimizing the ruin probability maximizes the adjustment coefficient in the classical ruin model with prevention, as well as the expected dividends until ruin in the model with dividends. We also show that the optimal prevention strategy is different if one aims at maximizing the average surplus at a fixed time horizon. A sensitivity analysis is carried out. We also prove that our results can be extended to the case where prevention starts to work only after a minimum prevention level threshold. We then study the case where prevention only works for severe claims. At the end of the talk, we shall explain how to target prevention towards the right policyholders in health insurance using some data analytics techniques.

Joint lifetime modelling with mIPH distributions

Hansjörg Albrecher[1], **Martin Bladt**[2] **and Alaric J.A. Müller**[3]

22 Aug
PS1

[1]*University of Lausanne, Switzerland, hansjoerg.albrecher@unil.ch*
[2]*University of Lausanne, Switzerland, martin.bladt@unil.ch*
[3]*University of Lausanne, Switzerland, alaric.mueller@unil.ch*

Keywords: mortality modelling, multivariate PH distributions, censoring, EM algorithm

Acyclic phase-type (PH) distributions have been a popular tool in survival analysis, thanks to their natural interpretation in terms of ageing towards its inevitable absorption. It is therefore interesting to consider the potential of multivariate PH distributions for the modelling of joint human lifetimes. In the univariate case, it was recently demonstrated in [2] that introducing time-inhomogeneity into the stochastic construction of the PH distribution can greatly reduce the number of needed dimensions for an adequate fit of mortality rate curves.

In this paper, we will consider an extension to the bivariate setting for the modelling of joint lifetimes. In contrast to previous models in the literature that were based on separate estimation of the marginal behavior and the dependence structure through a copula, using a new time-inhomogeneous version of a multivariate PH class (mIPH) we show how to model joint lifetimes without separating the estimation of marginal and dependence properties. This also leads to a more natural causal interpretation of the resulting model. We provide additional attributes of the mIPH class and an adapted estimation procedure that allows for right-censoring and covariate information. We show that initial distribution vectors can be tailored to reflect information that may affect the dependence of random variables, using multinomial regressions to predict the influence of covariates on starting probabilities. Moreover, we highlight the flexibility and parsimony in terms of needed phases that is introduced by time-inhomogeneity. We illustrate our results on the famous dataset of joint lifetimes of [7], where 10 phases turn out to be sufficient for a reasonable fitting performance. We finally interpret the results in terms of a correlated ageing mechanism of joint lifetimes that goes beyond a statistical fit.

References

[1] Albrecher, H. and Bladt, M. (2019). Inhomogeneous phase-type distributions and heavy tails. *Journal of Applied Probability* 56(4), 1044–1064.

[2] Albrecher, H., Bladt, M., Bladt, M. and Yslas, J. (2022+). Mortality modeling and regression with matrix distributions. *tPreprint, University of Lausanne. arXiv:2011.03219*.

[3] Asmussen, S., Nerman, O. and Olsson, M. (1996). Estimation of Phase-Type Distributions from Censored Data. *Scandinavian Journal of Statistics* 23(4), 419–441.

[4] Bladt, M. (2022). A tractable class of Multivariate Phase-type distributions for loss modeling: Theoretical developments. *Preprint, University of Lausanne. arXiv:2110.05179*.

[5] Boquan, C., Bruce, J., Liu, X. and Ren, J. (2021). The Mathematical Mechanism of Biological Aging. *North American Actuarial Journal* 25(1), 73–93.

[6] Dufresne, F., Hashorva, E., Ratovomirija, G. and Toukourou, Y. (2018). On age difference in joint lifetime modelling with life insurance annuity applications. *Annals of Actuarial Science* 12(2), 350–371.

[7] Frees, E.W., Carriere, J. and Valdez, E. (1996). Annuity Valuation with Dependent Mortality. *The Journal of Risk and Insurance* 63(2), 229–261

[8] Ji, M. and Hardy, M. and Li,J. S.-H. (2011). Markovian Approaches to Joint-Life Mortality. *North American Actuarial Journal* 11(4), 92–109.

[9] Olsoon, M. (1996). Estimation of Phase-Type Distributions from Censored Data. *Scandinavian Journal of Statistics* 23(4), 443–460.

Diversity of bivariate concordance measures

Martynas Manstavičius

Vilnius University, Lithuania, martynas.manstavicius@mif.vu.lt

23 Aug
PS5

Keywords: Scarsini axioms, polynomial-type concordance measures, copulas, transformation, multiplicative function

In this talk we will revisit Scarsini axioms for bivariate concordance measures, defined in terms of copulas (e.g., Spearman's ρ, Kendall's τ, Blomqvist's β, Gini's γ, etc.), and we will discuss several constructions of the so-called polynomial-type concordance measures. Constructed examples will show that the set of concordance measures is diverse, which will motivate further search for answers of several decade-old M. Taylor's questions (see, e.g., [1] and the references therein) about the characterization of degree $k \geq 2$ polynomial-type concordance measures in the bivariate, as well as multivariate, setting.

The talk is based on our recent work [2].

References

[1] M.D. Taylor (2016). Multivariate measures of concordance for copulas and their marginals, *Depend. Model.* 4, 224–236.

[2] M. Manstavičius (2022). Diversity of bivariate concordance measure, *MDPI Mathematics*, 10, 1103, `https://doi.org/10.3390/math10071103`

Exchangeable FGM copulas

Christopher Blier-Wong[1], **Helene Cossette**[2] **and Etienne Marceau**[3]

23 Aug
PS5

[1]*Université Laval, Canada, christopher.blier-wong.1@ulaval.ca*
[2]*Université Laval, Canada, helene.cossette@act.ulaval.ca*
[3]*Université Laval, Canada, etienne.marceau@act.ulaval.ca*

Keywords: copulas, stochastic representation, extreme points, exchangeable distributions

Copulas are a powerful tool to model dependence between the components of a random vector. One well-known class of copulas when working in two dimensions is the Farlie-Gumbel-Morgenstern (FGM) copula since their simple analytic shape enables closed-form solutions to many problems in applied probability. However, the classical definition of high-dimensional FGM copula does not enable a straightforward understanding of the effect of the copula parameters on the dependence, nor a geometric understanding of their admissible range. We circumvent this issue by studying the FGM copula from a probabilistic approach based on multivariate Bernoulli distributions. This paper studies high-dimensional exchangeable FGM copulas, a subclass of FGM copulas. We show that dependence parameters of exchangeable FGM can be expressed as convex hulls of a finite number of extreme points and establish partial orders for different exchangeable FGM copulas (including maximal and minimal dependence). We also leverage the probabilistic interpretation to develop efficient sampling and estimating procedures and provide a simulation study. Throughout, we discover geometric interpretations of the copula parameters that assist one in decoding the dependence of high-dimensional exchangeable FGM copulas.

On fair pricing via regularization

Lorenzo Marchi[1], **Katrien Antonio**[1,2], **Benjamin Avanzi**[3], **Rui Zhou**[3]

[1] *KU Leuven, Belgium, lorenzo.marchi@kuleuven.be*
[2]*University of Amsterdam, The Netherlands, katrien.antonio@kuleuven.be*
[3]*University of Melbourne VIC 3010, Australia, b.avanzi@unimelb.edu.au*

23 Aug
PS6

Keywords: Fairness, discrimination-free, insurance pricing, algorithmic bias, regularization, dependence measures

Recent advances in machine learning allow for the development of complex models that achieve high predictive accuracy and reveal latent societal inequalities [1]. In the actuarial context, technical pricing of insurance products via risk classification is an essential task. The pricing of risks according to risk levels is referred to as discrimination.

When considering factors to discriminate on, actuaries need to balance statistical accuracy with legal and regulatory constraints, amongst a number of considerations [2]. In particular, insurance companies may not be allowed to price according to certain criteria—protected classes—in certain jurisdictions (e.g., gender in Europe; see [3, 4]), even indirectly. This leads to so-called discrimination-free pricing, whereby prices are constructed so as to avoid the unwanted discrimination. Research on that problem has gathered considerable momentum, both using traditional statistical models and machine learning techniques [5, 6].

We propose a novel model-agnostic in-processing regularization approach to tackle algorithmic bias and ensure fair estimates with regard to protected classes. Moreover, our methodology is easily extendable to account for multiple protected features simultaneously with a minimal increase in model complexity. Results are illustrated and contrasted with existing methodologies [5, 6, in particular].

References

[1] Dressel, J., & Farid, H. (2018). The accuracy, fairness, and limits of predicting recidivism. *Science Advances*, 4(1), DOI:10.1126/sciadv.aao5580.

[2] Werner, G., & Modlin, C. (2010). Basic ratemaking. In *Casualty Actuarial Society*, Vol. 4, pp. 1-320.

[3] Council of European Union. 2004. Directive (EU) 2004/113/EC of 13 December 2004 implementing the principle of equal treatment between men and women in the access to and supply of goods and services. *Official Journal of European Union L* 373, 37-43.

[4] Council of European Union. 2012. Guidelines on the application of council directive 2004/113/EC to insurance, in the light of the judgment of the court of justice of the European Union in case C-236/09 (Test-Achats). *Official Journal of the European Union* 55, 1-11.

[5] Lindholm, M., Richman, R., Tsanakas, A., & Wüthrich, M. V. (2022). Discrimination-free insurance pricing. *ASTIN Bulletin: The Journal of the IAA* 52(1), 55-89.

[6] Grari, V., Charpentier, A., Lamprier, S., & Detyniecki, M. (2022). A fair pricing model via adversarial learning. Mar. 7, 2022. arXiv: 2022.12008v2.

On the impact of dependences and constraints in the optimal reinsurance treaty

23 Aug
PS6

Alexandra Bugalho de Moura[1] **and Manuel Guerra**[1]

[1]*ISEG - Lisbon School of Economics and Management, Universidade de Lisboa*
Research in Economics and Mathematics and Center for Applied Mathematics and Economics
amoura @iseg.ulisboa.pt, mguerra @iseg.ulisboa.pt

Keywords: reinsurance, dependent risks, Lipschitz constraint, premium calculation principles, expected utility

The optimal reinsurance problem of several dependent risks is analysed from the point of view of the direct insurer. The optimal treaty maximizing the expected utility is sough assuming independent negotiation of reinsurance for each risk, including different premium calculation principles for the different risks. The dependence structure of the risks is general and the reinsurance premium principles are moment based. The problem is studied considering both the presence or not of constraints. Example of a constraint is the Lipschitz constraint, usually imposed to avoid moral hazard. Through the analysis of optimality conditions, deduced in M.Guerra and A.B. Moura, Reinsurance of multiple risks with generic dependence structure (2021), and further developed to include constraints, a fixed point numerical scheme is devised to numerically obtain the optimal treaties of the general dependent risks with or without constraints. Numerical examples are presented and comparisons of the optimal solutions for dependent risks under constraints and with no constraints are performed.

Calculation of risk measures of variable annuity portfolios using neural network

22 Aug
PS3

Hang Nguyen[1], **Michael Sherris**[1], **Andres Villegas**[1] **and Jonathan Ziveyi**[1]

[1]*University of New South Wales, Sydney, Australia*

Keywords: variable annuity portfolio, risk measures, Monte-Carlo simulation

Stochastic reserving and calculation of capital requirements for variable annuity (VA) exposures require providers to calculate risk measures such as Value at Risk and Conditional Tail Expectation. In this paper, we propose a numerical method to speed up the stochastic simulation for the calculation of risk measures of a VA portfolio at time zero. Our proposed method is therefore only developed for a stochastic simulation that is not nested, however we expect that extension to nested simulation is possible. Even though the method is specifically targeted at the calculation of risk measures for a VA portfolio, it could also be generalized to any portfolio of assets requiring stochastic simulation. If we define $l_0(P_i, \Omega_j)$ to be the net liability of policy P_i at scenario Ω_j, then the aggregated net liability of the whole portfolio at scenario Ω_j will be:

$$L_0(\Omega_j) = \sum_{i=1}^{N} l_0(P_i, \Omega_j),$$

and the risk measures are:

$$VaR_\alpha := \inf \{y : \mathbb{P}[L_0 \leq y] \geq \alpha\},$$

$$\text{CTE}_\alpha := \mathbb{E}[L_0 \mid L_0 > VaR_\alpha].$$

Our proposed method uses an emulation framework, where we replace the calculation of the contract's net liability $l_0(P_i, \Omega_j)$ with an approximation $f(\cdot)$, such that $l_0(P_i, \Omega_j) \approx f(Z(P_i), \boldsymbol{\xi_j})$ where $Z(P_i)$ is the input of policy P_i and $\boldsymbol{\xi_j}$ is the input from the equity market that corresponds to the market condition at scenario Ω_j. Function f, called an emulator, is a surrogate model constructed by fitting a neural network model over a training set consisting of net liabilities of some contracts at a few selected scenarios. The equity market input $\boldsymbol{\xi_j}$ is a large input consisting of the trajectories of multiple financial assets over a long time horizon, so we use Multivariate Functional Principal Component Analysis to reduce its dimension. In our emulation framework, Monte Carlo simulation is carried out normally, however because the surrogate model can estimate a contract's net liability faster than exact calculations can compute, it runs faster than normal Monte Carlo simulation. Moreover, we can use the proposed framework quite efficiently to single out scenarios that most likely belong to the tail region, and since calculation of risk measures only requires tail events this leads to an obvious reduction in computational time compared to brute-force Monte Carlo simulation.

Model of lifetable evolution
Part 1: Extracting calendar and cohort effects form the data

23 Aug
PS5

Wojciech Otto

University of Warsaw, Poland, wotto@wne.uw.edu.pl

Keywords: longevity, Lee-Carter model, Haberman-Renshaw model, autoregression of residuals

The aim is to present selected issues arising when looking for an adequate model for long-term predictions of lifetables. Experience comes from analysis of Polish national lifetables for the period 1958-2020, and ages 0-94.

At the first stage Lee-Carter-type models have been fitted to the data. For each gender three calendar effects have been extracted, representing evolution of mortality of separated (partly) age groups: the young, the adult, and the old. Additionally, the restricted cohort effect has been included, too. Apart of the global effects (calendar and cohort) a kind of "local" effects have been captured through autoregression of residuals. Autoregression function encompasses as well residuals from some adjacent ages as well as the current and some past years. Such two-dimesional autoregression resembles a little bit specifications in some spatial models, however, the similarity is limited.

Parameters have been estimated by ML, assuming t-distribution of residuals, and imposing several hard and soft restrictions.

Results confirm that essential long-term behavior is reflected sufficiently by calendar effects. Global cohort effects as well as autocorrelation, possibly important for predictions within short or moderate horizon, could be neglected when long-run trends are of interest.

Model of lifetable evolution
Part 2: Time series model with stochastically varying drift

23 Aug
PS5

Wojciech Otto

University of Warsaw, Poland, wotto@wne.uw.edu.pl

Keywords: longevity, Haberman-Renshaw model, stochastically varying parameters, Kalman filter, cointegration

This part of the research focus on looking for a 6-dimensional time series model, good enough to capture long-term evolution of calendar effects that have been extracted by the Lee-Carter type of model, described within Part 1 of the presentation. Analyzed time series show that slope of trends is not stable. This is especially evident for adult males with the dramatic change of slope around 1991, however, one can observe symptoms of similar trend changes also for adult females. Some symptoms of trend changes could be observed for other age groups and other time epochs, as well.

The instability of trends raises the question of meaningfulness of models designed to produce long-run projections, such as these that we use for analyzing stability of pension systems or risk of long-term life insurance. It seems that a proper model should incorporate this instability into its specification. We have to accept that point predictions are burdened by large errors. Perhaps the best thing to do is to focus on measuring the scope of uncertainty. This may lead to designing pension and life products that are more flexible, perhaps leaving some part of risk on the side of pensioners/insureds.

Technically, the presentation focuses on criteria for model selection suitable when the aim is long-run projections. After setting a list of desired properties of long-run behavior, a range of admissible specifications has been set. Within this range selection might be based to a larger extent on goodness of fit and related criteria, traditionally used for model selection by econometricians.

The results confirmed that:

- changes of mortality observed within all 6 subgroups young, adult, old×males, females are strongly interrelated – which concerns as well trend changes as deviations
- the slope of trends varies in time significantly,
- errors of long-term prediction are dramatically larger for models with proper specification as compared to some “naive” models that ignore stochastic variations of trends.

Premium control with reinforcement learning

Lina Palmborg

Stockholm University, Sweden, lina.palmborg@math.su.se

24 Aug
PS7

Keywords: premium control, reinforcement learning, Markov decision process

We consider a premium control problem in discrete time, inspired by [1] and [2], formulated in terms of a Markov decision process. In a simplified setting, the optimal policy can be derived with dynamic programming methods. However, these classical methods are not feasible in a more realistic setting due to the dimension of the state space. Hence, to combat the curse of dimensionality we explore reinforcement learning techniques, using linear function approximation, see e.g. [3]. We illustrate the appropriateness of the approximate optimal policy compared with the true optimal policy in a simplified setting, and further demonstrate that the approximate optimal policy outperforms benchmark policies in a more realistic setting where classical approaches fail.

This is based on joint work with F. Lindskog.

References

[1] Martin-Löf, A. (1983). Premium control in an insurance system, an approach using linear control theory, *Scandinavian Actuarial Journal*, 1983:1, 1-27, DOI: 10.1080/03461238.1983.10408686

[2] Martin-Löf, A. (1994). Lectures on the use of control theory in insurance, *Scandinavian Actuarial Journal*, 1994:1, 1-25, DOI: 10.1080/03461238.1994.10413927

[3] Sutton, R. S., Barto, A. G. (2018). *Reinforcement learning: An introduction*. MIT Press.

Tails of moments of sums with heavy-tailed summands and applications to the Haezendonck-Goovaerts risk measure

22 Aug
PS3

Remigijus Leipus[1], **Saulius Paukštys**[2], **Jonas Šiaulys**[3]

[1]*Vilnius university, Lithuania, remigijus.leipus@mif.vu.lt*
[2]*Vilnius university, Lithuania, saulius.paukstys@mif.vu.lt*
[3]*Vilnius university, Lithuania, jonas.siaulys@mif.vu.lt*

Keywords: tail moment, sum of random variables, heavy tailed distributions, dominatedly varying distributions, Haezendonck-Goovaerts risk measure

We consider the sum $S_n^\xi := \xi_1 + ... + \xi_n$ of possibly dependent real-valued random variables $\xi_1, ..., \xi_n$ with heavy tailed distributions. We are interested in asymptotic behaviour of the tail moment $\mathbb{E}((S_n^\xi)^\alpha \mathbb{1}_{\{S_n^\xi > x\}})$, where α is a nonnegative real number. Our main objective is to asymptotically bound this tail moment by the sums of individual tail moments $\mathbb{E}((\xi_k)^\alpha \mathbb{1}_{\{\xi_k > x\}})$ with some specific correcting constants. We apply the obtained results by deriving asymptotic estimations for the Haezendonck-Goovaerts risk measure of sums with dominatedly varying increments.

Homogeneous approximations of time-inhomogeneous semi-Markov life insurance models

22 Aug
PS3

Oscar Peralta

University Lausanne, Switzerland, oscar.peraltagutierrez@unil.ch

Keywords: strong convergence, time-inhomogeneous Markov jump processes, uniformization

A (time-inhomogeneous) semi-Markov model is a multi-state process whose jump probabilities are dependent on the current state, elapsed time, and duration since the last jump. In the context of life insurance contracts, semi-Markovian models allow for transitions that are not only age-dependent, but also dependent on the time elapsed since the person last transitioned between states; see [1] and references therein. In this talk we show how semi-Markovian models can be approximated arbitrarily well, in a pathwise sense, by a tractable class of time-homogeneous Markov jump processes, which are constructed by observing the original semi-Markov model at an increasingly dense set of Poisson arrivals. Our approximation opens the window to study descriptors of semi-Markovian life insurance models (for example, their transition probabilities) by means of homogeneous Markov jump processes. This work is partly based in [2].

References

[1] Buchardt, K., Møller, T., Schmidt, K. B. (2015). Cash flows and policyholder behaviour in the semi-Markov life insurance setup. *Scandinavian Actuarial Journal* 8, 660-688.

[2] Bladt, M., Peralta, O. (2022). Strongly convergent homogeneous approximations to inhomogeneous Markov jump processes and applications. *arXiv preprint*, *arXiv:2204.02954*.

Actuarial calculations for reserve-dependent payments in life insurance under information shrinkage

22 Aug
PS2

Jannes Tjark Rastedt

University of Oldenburg, Germany, jannes.tjark.rastedt@uni-oldenburg.de

Keywords: life insurance, non-monotone information, information restriction, non-linear BSDEs

In life insurance, the discounted cumulative future payments of an insurance contract are of central interest, but they are usually unknown at present. Given the available information, conditional expectations are considered instead, enabling the insurer to calculate the so-called prospective reserve.

We extend the insurance model by considering contractual payments that may depend non-linearly on the prospective reserve and we allow for non-monotone and restricted information structures. In the existing literature, see for example [1], a filtered probability space and the application of martingale theory are central to showing existence and uniqueness of solutions to the corresponding BSDE formulation of the prospective reserve. These martingale methods do not work under information shrinkage, so the infinitesimal martingale concept, introduced in [2], is used instead.

In my talk, I use the fixed-point theorem to show existence and uniqueness of the non-adapted payment process itself, before extending the results to the prospective reserve and the calculation of a net equivalent premium.

Applications of the theory include legal restrictions, like the 'right to erasure' (General Data Protection Regulation 2016/679), and model simplifications, like unisex-tariffs.

References

[1] Christiansen, M.C., Djehiche, B. (2020). Nonlinear reserving and multiple contract modifications in life insurance. *Insurance: Mathematics and Economics* 93, 187–195.

[2] Christiansen, M.C. (2021). Time-dynamic evaluations under non-monotone information generated by marked point processes. *Finance and Stochastics* 25, 563–596.

Dealing with mortality shocks in a stochastic multi-population mortality model

22 Aug
PS1

Jens Robben[1] **and Katrien Antonio**[1,2]

[1]*KU Leuven, Belgium, jens.robben@kuleuven.be; katrien.antonio@kuleuven.be*
[2]*University of Amsterdam, The Netherlands*

Keywords: mortality shocks, multi-population mortality model, stochastic mortality modelling, calibration, forecasting, Li and Lee model

The gradual improvement in mortality rates over the last few decades is halted in 2020-2021 by a mortality shock produced by the COVID-19 pandemic. Other pandemics and epidemics (e.g. cholera, Spanish flu) or wars have caused similar shocks in the less recent past. We aim to assess the impact of mortality shocks on the calibration of a stochastic multi-population mortality projection model of type Li and Lee [5] and its resulting projections for future mortality rates. This type of mortality model has become a standard for projecting mortality in Belgium and the Netherlands [1, 4]. We augment the Li and Lee mortality model with an age-dependent mortality jump component to account for potential mortality shocks in future mortality rate projections. Adding an age-dependent regime-switching mortality process [2, 7] or an age-dependent transitory jump process [6] to the Li and Lee mortality model are two possible options. We calibrate this mortality model on annual death counts and exposures at the level of individual ages. This type of mortality data are typically collected, produced and reported with a significant delay of - for some countries - several years on a platform such as the Human Mortality Database [3]. To enable a timely evaluation of the impact of the COVID-19 pandemic data points in 2020-2021, we have to rely on other data sources (e.g., the Short-Term Mortality Fluctuations Data series) that swiftly publish weekly mortality data collected in age buckets. To be compliant with the design and calibration strategy of the Li and Lee model, we transform the weekly mortality data collected in age buckets to yearly, age-specific observations using our proposed protocol [8]. To summarize, our research provides a toolbox for simulating various mortality scenarios, both with and without shocks, and allows us to quantify the impact on future mortality rates and the valuation of life contingent risks.

References

[1] Antonio, K., Devriendt, S., Robben, J. (2020). *The IA|BE 2020 mortality projection for the Belgian population*. Available online: `https://katrienantonio.github.io/papers/report_iabe_2020.pdf`

[2] Hainaut, D. (2012). Multidimensional Lee–Carter model with switching mortality processes. *Insurance: Mathematics and Economics* 50(2), 236–246.

[3] *Human Mortality Database*. University of California, Berkeley (USA); and Max Planck Institute for Demographic Research (Germany), available online: `www.mortality.org`.

[4] Koninklijk Actuarieel Genootschap (2020). *Projections life table AG2020*. Available online: `https://www.ag-ai.nl/view/46337-PrognosetafelAG2020-ENG-print.pdf`

[5] Li, N., Lee, R. (2005). Coherent mortality forecasts for a group of populations: an extension of the Lee–Carter method. *Demography* 42, 575–94.

[6] Liu, Y., Li, J.S.H. (2015). The age pattern of transitory mortality jumps and its impact on the pricing of catastrophic mortality bonds. *Insurance: Mathematics and Economics* 64, 135–150.

[7] Milidonis, A., Lin, Y., Cox, S.H. (2011). Mortality regimes and pricing. *North American Actuarial Journal* 15(2), 266–289.

[8] Robben, J., Antonio, K., Devriendt, S. (2022). Assessing the Impact of the COVID-19 Shock on a Stochastic Multi-Population Mortality Model. *Risks* 10(2), 26.

Impact of correlation between interest rates and mortality rates on the value of a zero-coupon survival bond

22 Aug
PS1

Benjamin Roelants du Vivier[1], **Griselda Deelstra**[1] **and Pierre Devolder**[2]

[1]*Université Libre de Bruxelles, Belgium, griselda.deelstra@ulb.be*
[2]*Université Catholique de Louvain, Belgium, pierre.devolder@uclouvain.be*

Keywords: stochastic mortality, Hulland White[2], correlation between mortality and financial markets, zero-coupon survival bond, affine processes

In this paper we establish the expression of the best estimate of a zero-coupon survival bond when modeling the interest rates and the mortality rates with two Hull and White models correlated to each other, meaning that we relax the traditional assumption of independence between mortality risk and interest rate risk, like in [1].

We end up with a very elegant multiplicative structure in which the price of the zero-coupon survival bond with correlation is expressed as the price without correlation times a factor representing the correlation impact.

After calibration based on [2], we investigate the impact of the inclusion of correlation on the price of a zero-coupon survival bond from different perspectives.

First we perform sensitivity analysis on the price of correlation with the aim of identifying to what extent each of the parameters appearing in the price of correlation formula influences the final value. The key outcome is that when some of the aforementioned parameters (especially the value of maturity) take values in the upper part of their range, the impact of correlation can become very significative and has to be taken into account!

Second, leaving the survival probability apart, we can write the yield to maturity of the zero-coupon survival bond with correlation, as the sum of the yield to maturity without correlation and the yield to maturity of correlation. We then perform the same kind of sensitivity analysis.

Finally, we compute the price of annuities with and without correlation and conclude.

References

[1] Devolder, P. and Azizieh, C. (2013). Margrabe Option and Life Insurance with participation. *Bulletin d'actuariat français* 13, 47-77.

[2] Zeddouk, F. and Devolder, P. (2020). Mean reversion in stochastic mortality: why and how? *European Actuarial Journal* 10, 499-525.

IBNR and RBNS models in multi-state life insurance

Kristian Buchardt[1], **Christian Furrer**[2] **and Oliver Lunding Sandqvist**[1,2]

22 Aug
PS3

[1]*PFA Pension, Copenhagen, Denmark*
[2]*University of Copenhagen, Denmark*

Keywords: transaction time, valid time, claims reserves, prospective reserves, piecewise deterministic processes

Benefits and premiums in life insurance contracts are typically paid contingent on the biometric state of the insured. Due to delays between the occurrence, reporting, and settlement of changes to the biometric state, the state process is not fully observable in real-time. This fact implies that the popular multi-state models for the biometric state of the insured are not able to describe the development of the policy in real-time.

When calculating prospective reserves, one therefore has to account for the fact that the currently observed biometric state might be changed retroactively due to the arrival of new information. This phenomenon gives rise to the need for incurred-but-not-reported (IBNR) and reported-but-not-settled (RBNS) models, which are currently not well described in the context of multi-state life insurance models.

We highlight the need for a unifying approach. Subsequently, we give a fundamental treatment of the problem in the setting of continuous-time multi-state life insurance by introducing a new class of models: transaction time models. Furthermore, when specializing to disability insurance schemes, we derive an explicit formula for the IBNR and RBNS reserves.

Modeling implied volatility surfaces of crypto options

Artur Sepp[1] **and Parviz Rakhmonov**[2]

[1]*Sygnum Bank, Switzerland, artursepp@gmail.com*
[2]*Citi Bank, UK, parviz.msu@gmail.com*

22 Aug
PS1

Keywords: stochastic volatility modeling, crypto options, martingale measures

Modelling of implied volatility surfaces of crypto assets, such as Bitcoin and Ether, is challenging when applying conventional stochastic volatility (SV) models. On the theoretical side, the dominance of inverse pay-off functions requires stronger conditions for the existence of equivalent martingale measures. On the practical side, the positive correlation between the price-returns and volatility and price may either invalidate some SV models or pose tight restrictions on model parameters.

First, we introduce a market model for the valuation of vanilla and inverse options on crypto assets under the equivalent martingale money market account measure and the inverse spot measure. We derive conditions for the existence of such valuation measures under conventional SV models and we show limitations of popular SV models when the correlation between returns and their volatility dynamics is positive.

Second, we introduce the lognormal SV model with the quadratic drift for arbitrage-free dynamics of implied volatilities. Despite the non-linear drift, we show that the volatility process has a strong solution. We derive the domain model parameters for which the equivalent spot and inverse measures exists.

We then develop an analytic approach for valuation of options on price and quadratic variance based on a new method of affine expansion for the moment generation function of price processes with non-affine and non-linear dynamics. We proof the stability of the expansion method and demonstrate its accuracy in option pricing applications.

We finally illustrate the application of the lognormal SV model for calibrating and modeling of Bitcoin and Ether volatility surfaces using Deribit options exchange data.

References

[1] Sepp A., Rakhmonov P. (2022). Log-Normal Stochastic Volatility Model with Quadratic Drift: Analytic Approach with Applications to Crypto Options, *working paper*

Determinants of institutional long-term care of dependent elderly in Switzerland

24 Aug
PS7

Martin Bladt[1], **Michel Fuino**[2], **Aleksandr Shemendyuk**[3] **and Joël Wagner**[4]

[1]*University of Lausanne, Switzerland, martin.bladt@unil.ch*
[2]*Retraites Populaires, Switzerland, fuinom@gmail.com*
[3]*University of Lausanne, Switzerland, aleksandr.shemendyuk@unil.ch*
[4]*University of Lausanne, Switzerland, joel.wagner@unil.ch*

Keywords: long-term care, institutional care, accelerated failure time, beta regression, empirical data

The financing of long-term care causes intense interest due to demographic changes and aging population in many countries. While many care-intensive conditions begin to manifest at higher ages, the assessment of the financial costs, the infrastructure and the number of qualified personnel becomes a necessity. In order to evaluate these costs and needs, the overall burden of institutional care can be derived from the duration of stay in dependence and the intensity of help provided to the elderly for their activities of daily living. In this research, we aim to model these two aspects using novel longitudinal data from nursing homes in the canton of Geneva in Switzerland. Our data contains comprehensive health and care information, including physical and psychological impairments, levels of dependence, and pathologies, on about 21 000 individuals. On the one hand, we build an accelerated failure time model to study the influence of selected factors on the duration of care. On the other hand, a beta regression model describes the intensity of care. Our main findings show that the duration, apart from age and gender, is mainly affected by the pathologies and number of different diagnoses, while the intensity of care is driven by the individual level of dependence and specific limitations. Using both evaluations, we simulate the total care burden for individual profiles. Our study sheds light on the relevant physical and psychological health indicators that need to be accounted for, not only by the providers of institutional care, but also, e.g., by the government when setting up care policies, and by insurance companies for the pricing of long-term care cover.

Study of institutionalized elderly profiles derived from multiple health factors

24 Aug
PS7

Aleksandr Shemendyuk[1] **and Joël Wagner**[2]

[1]*University of Lausanne, Switzerland, aleksandr.shemendyuk@unil.ch*
[2]*University of Lausanne, Switzerland, joel.wagner@unil.ch*

Keywords: long-term care, institutional care, spectral clustering, multinomial regression, empirical data

Due to advanced medicine and increased quality of life, people live longer. In many developed countries the population aging comes with a series of issues related to the organization and the financing of long-term care. The determinants of the overall institutionalized care burden are well-studied in previous works and, thus, the understanding of the expenses is achieved. On the other hand, the organization of the institutionalized long-term must satisfy the requirements of both institutions and elderly. One way to optimize the management is to leverage the information on health issues of the elderly to assess the required qualifications of the personnel and the allocation of newcomers. In this research we aim to determine the typical health profiles of institutionalized elderly using novel longitudinal data from nursing homes in the canton Geneva in Switzerland. Our data contains comprehensive information of the health factors such as psychological and sensory functions impairments, levels of limitations, and pathologies, on about 18 000 individuals covering the period from 1996 to 2018. First, we perform a spectral clustering algorithm and determine typical health profiles of the institutionalized elderly. Then, using a multinomial logistic regression we study the effects of the health factors that determine the health profiles. Our main findings show that there are eight typical profiles, the biggest of which consists of the most “healthy” elderly who on average require the least amount of help for their daily needs and who stay in the institution the longest. We show that, unlike the age at entry and the gender, the limitations and the set of pathologies are relevant factors in determining the health profile. Our study sheds light on the typical structures of elderly health profiles, which can be used by the institutions to organize infrastructure and human resources as well as by the insurance companies to derive profile-based products bringing additional insurance coverage in case of needs and in line with elderlies’ health trajectories.

Development of a tool for making projections in DC pension schemes

23 Aug
PS4

Frederico Pinheiro[1], **Sofia Frederico**[1] **and Onofre Simões**[2]

[1]*ASF, Portugal, frederico.pinheiro@asf.com.pt, sofia.frederico@asf.com.pt*
[2]*ISEG-ULisboa, Portugal, onofre@iseg.ulisboa.pt*

Keywords: European Union, retirement income, DC schemes, ESG, real-world valuation, projected lifetable

The ageing of the population is leading to reforms in Social Security systems with negative impact on the levels of retirement income. One way to minimize this impact is to reinforce the role of complementary pension schemes. Pension projections can be an important tool to support individuals in their decision-making process about saving for retirement and have been a part of several initiatives at the EU level.

This work focuses on the development of a calculation tool for making pension projections in the scope of occupational DC schemes. We aim to study the potential performance of different investment strategies using an Economic Scenario Generator framework and to evaluate the impact on the retirement income that such strategies will produce, considering also different assumptions with regard to mortality and interest. The model developed considers three main risk factors: (1) financial market risk, which includes uncertainty over return on investments, inflation and interest rates; (2) labour risk, originated from uncertainty over the real wage growth path; (3) demographic risk, as a result of increasing life expectancy.

References

[1] Antolin, P., Payet, S. (2010). Assessing the Labour, Financial and Demographic Risks to Retirement Income from Defined-Contribution Pensions. *OECD Journal Financial Market Trends* 2010(2), 8-8.

[2] Bosworth, B., G. Burtless, and C. E. Steuerle, (2000). Lifetime Earnings Patterns, the Distribution of Future Social Security Benefits, and the Impact of Pension Reform. *Social Security Bulletin*, 63(4), 74-98.

[3] Directive (EU) 2016/2341 of the European Parliament and of the Council of 14 December 2016 on the activities and supervision of IORPs.

[4] EIOPA (2018). Report on the pension benefit statement: Guidance and principles based on current practices.

[5] EIOPA (2019). Consultation paper on the proposed approaches and considerations for EIOPA´s technical advice, implementing and regulatory technical standards under Regulation (EU) 2019/1238 on a PEPP.

[6] EIOPA (2020). PEPP: EIOPA'S stochastic model for a holistic assessment of the risk profile and potential performance.

The Gerber-Shiu discounted penalty function for the bi-seasonal discrete time risk model

22 Aug
PS1

Jonas Sprindys

Vilnius University, Lithuania, jonas.sprindys@mif.vu.lt

Keywords: bi-seasonal model, discrete time risk model, Gerber-Shiu function, penalty function, time of ruin

We consider the discrete time risk model with two seasons. In such model, the claims repeat with time periods of two units, i.e. claim distributions coincide at all even instants and at all odd instants. Our purpose is to derive an algorithm for calculating the values of the particular case of the Gerber-Shiu discounted penalty function $\mathbb{E}\left(\mathrm{e}^{-\delta T}\mathbb{1}_{\{T<\infty\}}\right)$, where T is the time of ruin, and δ is a constant nonnegative force of interest. Theoretical results are illustrated by some numerical examples.

One-year and ultimate reserve risk in Mack Chain Ladder model

Marcin Szatkowski[1] **and Łukasz Delong**[2]

23 Aug
PS6

[1]*SGH Warsaw School of Economics, STU ERGO Hestia SA, Poland, marcin.szatkowski@sgh.waw.pl*
[2]*SGH Warsaw School of Economics, Poland, lukasz.delong@sgh.waw.pl*

Keywords: one-year risk, ultimate risk, reserve risk, emergence pattern, Mack Chain Ladder

We investigate the relation between one-year reserve risk and ultimate reserve risk in Mack Chain Ladder model in a simulation study. The first goal is to validate the so-called linear emergence pattern formula, which maps the ultimate loss to the one-year loss, in case when we measure the risks with Value-at-Risk. The second goal is to estimate the true emergence pattern of the ultimate loss, i.e. the conditional distribution of the one-year loss given the ultimate loss, from which we can properly derive a risk measure for the one-year horizon from the simulations of ultimate losses. Finally, our third goal is to test if classical actuarial distributions can be used for modelling of the outstanding loss from the ultimate and the one-year perspective. In our simulation study we investigate several synthetic loss triangles with various duration of the claims development process, volatility, skewness and distributional assumptions of the individual development factors. We quantify the reserve risks without and with the estimation error of the claims development factors.

Are long-term care insurance customers likely to come with greater risks for insurers? Reflections and simulations from a survey in Switzerland

24 Aug
PS7

Andrey Ugarte[1] **and Joël Wagner**[2]

[1]*University of Lausanne, Switzerland, andrey.ugarte@unil.ch*
[2]*University of Lausanne, joel.wagner@unil.ch*

Keywords: long-term care, insurance, dependence, information asymmetry, adverse selection

The long-term care insurance (LTCI) market in Switzerland is still in a very early development stage. In this work, we make use of a representative sample of the Swiss population to simulate the likely effects of previously discovered information asymmetries in the LTCI market. We build a model based on Monte Carlo simulations and provide estimations of the expected probability to lose autonomy in a lifetime and the duration of dependence. We do this by developing new findings on age preferences when purchasing LTCI and desired cover levels, as well as by resorting to previous findings' on LTCI preferences of potential customers, probabilities to lose autonomy by age and gender, and the factors affecting duration of dependence [1, 2, 3]. Thereby, we compare the dependence frequency and severity of the sub-population that has shown interest in LTCI with the general population in different mortality scenarios. Indeed, we find that, in the Swiss demographic context, individuals have a high probability to experience loss of autonomy in their lifetime. Moreover, through sensitivity analysis, we show that plausible mortality variations suffice to change the probability to lose autonomy by more than 3%. Consequently, we identify the population aged between 80 and 85 years as key for the demand for long-term care. From our analysis we do not find evidence to believe that those interested in LTCI coverage are so based on privileged information about them being at greater risk. In fact, by contrasting people's beliefs about their own chances to lose autonomy and our model's results, we show that most individuals are not aware of their risk to lose autonomy, which makes potential adverse selection in the LTCI market rather difficult.

References

[1] Fuino, M. and J. Wagner (2018). Long-term care models and dependence probability tables by acuity level: New empirical evidence from Switzerland. *Insurance: Mathematics and Economics* 81, 51-70.

[2] Fuino, M. and J. Wagner (2020). Duration of long-term care: Socio-economic factors, type of care interactions and evolution. *Insurance: Mathematics and Economics* 90, 151-168.

[3] Fuino, M., J. Wagner, and A. Ugarte Montero (2021). On the Drivers of Potential Customers' Interest in Long-Term Care Insurance: Evidence from Switzerland.

On the drivers of potential customers' interest in long-term care insurance: evidence from Switzerland

24 Aug
PS7

Michel Fuino[1], **Andrey Ugarte**[2] **and Joël Wagner**[3]

[1]*Retraites Populaires, fuinom@gmail.com*
[2]*University of Lausanne, Switzerland, andrey.ugarte@unil.ch*
[3]*University of Lausanne, joel.wagner@unil.ch*

Keywords: long-term care, dependence, care preferences, care costs

As the risks associated with aging start to materialize, societies become more aware of the financial consequences of long-term care (LTC). While limited coverage is available through social insurance in many countries, attractive offers of private products barely exist and a lack of knowledge about LTC insurance (LTCI) persists. Based on a novel survey on aging, health and dependence carried out in Switzerland, this research aims to comprehend the key drivers that make individuals interested in buying care insurance products for themselves in Switzerland. Using models that combine features from both classical statistics and machine learning techniques, we depict the characteristics of potential buyers based on key economic, social, demographic, and political factors. We find that factors relating to the awareness and understanding of LTC turn out to be extremely relevant. Further, we observe that self-perceived health, behavior, and trust relationships between customers and insurers are important. Socio-economic factors only play a secondary role in the decision-making process. Our findings are relevant beyond the academic community and for policymakers and private insurers alike.

Mortality/longevity risk-minimization with or without securitization

23 Aug
PS5

Tahir Choulli[1], **Catheirne Daveloose**[2] **and Michèle Vanmaele**[2]

[1]*University of Alberta, Edmonton, Canada, tchoulli@ualberta.ca*
[2]*Ghent University, Belgium, michele.vanmaele@ugent.be*

Keywords: time of death/random horizon/default, progressively enlarged filtration, optional martingale representation, risk decomposition, unit-linked mortality contracts, risk-minimization, mortality/longevity risk, insurance securitization

In this talk we will address the risk-minimization problem, with and without mortality securitization, à la Föllmer–Sondermann for a large class of equity-linked mortality contracts when no model for the death time is specified. This framework includes situations in which the correlation between the market model and the time of death is arbitrary general, and hence leads to the case of a market model where there are two levels of information—the public information, which is generated by the financial assets, and a larger flow of information that contains additional knowledge about the death time of an insured. We will derive the dynamics of the value processes of the mortality/longevity securities used for the securitization, and decompose any mortality/longevity liability into the sum of orthogonal risks by means of a risk basis. Next, we will quantify, as explicitly as possible, the effect of mortality on the risk-minimizing strategy by determining the optimal strategy in the enlarged filtration in terms of strategies in the smaller filtration. We will obtain risk-minimizing strategies with insurance securitization by investing in stocks and one (or more) mortality/longevity derivatives such as longevity bonds.

References

[1] Choulli, T., Daveloose, C., Vanmaele, M. (2020). A martingale representation theorem and valuation of defaultable securities. *Mathematical Finance* 20(4), 1527-1564.

[2] Choulli, T., Daveloose, C., Vanmaele, M. (2021). Mortality/longevity risk-minimization with or without securitization. *Mathematics* 9(14), 1629 (27 pages).

Red light, green light: Gaussian process regression to validate capped volatility swaps

23 Aug
PS6

Stephan Höcht[1], **Wim Schoutens**[2] **and Eva Verschueren**[3]

[1]*Assenagon GmbH, Germany, stephan.hoecht@assenagon.com*
[2]*University of Leuven, Belgium, wim.schoutens@kuleuven.be*
[3]*University of Leuven, Belgium, eva.verschueren@kuleuven.be*

Keywords: volatility, volatility swaps, pricing model, Gaussian process regression, implied moments

A volatility swap is a forward contract on an asset's annualized, realized volatility, over a fixed period of time [1]. At expiration, the payoff of the contract is given by

$$\text{Notional} \times \left[\min\left(\text{CapLevel}, \text{RealizedVolatility}\right) - \text{VolatilityStrikePrice}\right].$$

The cap level limits the risk exposure of the issuer of the contract and is most often fixed at 2.5 times the strike.

Volatility swaps are directly exposed to the volatility of the underlying asset, making volatility a tradable market instrument. For this reason, the contracts are nowadays popular tools in fund-based risk management and are used for both speculative and hedging purposes. Volatility swaps are traded over-the-counter, meaning that no price is readily available on exchange. However, fund managers can call upon external pricing entities to receive the current price of a specific contract. In reality, this price is often the end product of an unknown internal procedure. Moreover, occasionally, prices from different pricing sources differ substantially, which hampers the correct valuation of an investment fund.

In this presentation, we show how we can deploy machine learning techniques based on Gaussian process regression to price capped volatility swaps [2]. This model then serves as a validation tool for external prices of volatility swaps, assigning red lights to prices that are too distant from the predicted value.

To this purpose, we build a unique dataset consisting of daily observed prices of traded volatility swaps over the lifetime of the contract, on individual stocks as well as stock indices. Before settlement of the contract, the unknown component within the payoff structure is the realized volatility, meaning that especially features with predictive power for this future realized volatility need to be taken into account. To this end, information from the implied moments on the underlying asset is exploited, to complement other features such as strike, (remaining) time till maturity and accrued volatility up to the valuation date.

References

[1] Demeterfi, K., Derman, E., Kamal, M. and Zou, J. (1999). More Than You Ever Wanted To Know About Volatility Swaps. Quantitative Strategies Research Notes, Goldman Sachs, March, 1–52.

[2] Rasmussen, C. E. and Williams, C. K. I. (2006). *Gaussian Processes for Machine Learning*. MIT Press.

Ensemble distributional forecasting for insurance loss reserving

Benjamin Avanzi[1], **Yanfeng Li**[2], **Bernard Wong**[3] **and Alan Xian**[4]

24 Aug
PS7

[1]*University of Melbourne, Australia, b.avanzi@unimelb.edu.au*
[2]*University of New South Wales, Australia, yanfeng.li@student.unsw.edu.au*
[3]*University of New South Wales, Australia, bernard.wong@student.unsw.edu.au*
[4]*Macquarie University, Australia, alan.xian@mq.edu.au*

Keywords: aggregate loss reserving, ensemble learning, linear pool, distributional forecast

Loss reserving generally focuses on identifying a single model that can generate superior predictive performance. However, different loss reserving models specialise in capturing different aspects of loss data. This is recognised in practice in the sense that results from different models are often considered, and sometimes combined. For instance, actuaries may take a weighted average of the prediction outcomes from various loss reserving models, often based on subjective assessments.

In this paper, we propose a systematic framework to objectively combine (i.e. ensemble) multiple stochastic loss reserving models such that the strengths offered by different models can be utilised effectively. Criteria of choice consider the full distributional properties of the ensemble. A notable innovation of our framework is that it is tailored for the features inherent to reserving data. These include, for instance, accident, development, calendar, and claim maturity effects. Crucially, the relative importance and scarcity of data across accident periods renders the problem distinct from the traditional ensembling techniques in statistical learning.

Our ensemble reserving framework is illustrated with a complex synthetic dataset. In the results, the optimised ensemble outperforms both (i) traditional model selection strategies, and (ii) an equally weighted ensemble. In particular, the improvement occurs not only with central estimates but also relevant quantiles, such as the 75th percentile of reserves (typically of interest to both insurers and regulators).

Phase-type mixture-of-experts regression for loss severities

Martin Bladt[1] **and Jorge Yslas**[2]

23 Aug
PS5

[1]*University of Lausanne, Switzerland, martin.bladt@unil.ch*
[2]*University of Liverpool, UK, Jorge.Yslas-Altamirano@liverpool.ac.uk*

Keywords: severity modeling, phase-type distributions, mixture-of-experts models, EM algorithm

The task of modeling claim severities is addressed when data is not consistent with the classical regression assumptions. This framework is common in several lines of business within insurance and reinsurance, where catastrophic losses or heterogeneous sub-populations result in data difficult to model. Their correct analysis is required for pricing insurance products, and some of the most prevalent recent specifications in this direction are mixture-of-experts models. This talks proposes a regression model that generalizes the latter approach to the phase-type distribution setting. More specifically, the concept of mixing is extended to the case where an entire Markov jump process is unobserved and where states can communicate with each other. The covariates then act on the initial probabilities of such underlying chain, which play the role of expert weights. The basic properties of such a model are computed in terms of matrix functionals, and denseness properties are derived, demonstrating their flexibility. An effective estimation procedure is proposed, based on the EM algorithm and multinomial logistic regression, and subsequently illustrated using simulated and real-world datasets.

A collective reserving model with claim openness

Henning Zakrisson

Stockholm University, Sweden, zakrisson@math.su.se

23 Aug
PS6

Keywords: aggregated reserving, open claim dynamics, RBNS and IBNR reserves

The present paper introduces a simple aggregated reserving model based on claim count and payment dynamics, which allows for claim closings and re-openings. The modelling starts off from individual Poisson process claim dynamics in discrete time, keeping track of accident year, reporting year, and payment delay. This modelling approach is closely related to the one underpinning the so-called double chain-ladder model, and it allows for producing separate reported but not settled (RBNS) and incurred but not reported (IBNR) reserves. Even though the introduction of claim closings and re-openings will produce new types of dependencies it is possible to use flexible parametrisations in terms of, e.g., generalised linear models (GLM) whose parameters can be estimated based on aggregated data using quasi-likelihood theory. Moreover, it is possible to obtain interpretable and explicit moment calculations, as well as having consistency of normalised reserves when the number of contracts tend to infinity. Further, by having access to simple analytic expressions for moments it is computationally cheap to bootstrap the mean squared error of prediction for reserves.

Based on joint work with Mathias Lindholm, Stockholm University.

Optimal price structure of cyber insurance policies with risk mitigation services

23 Aug
PS4

Gabriela Zeller[1] **and Matthias Scherer**[2]

[1]*Technical University of Munich, Germany, gabi.zeller@tum.de*
[2]*Technical University of Munich, Germany, scherer@tum.de*

Keywords: cyber risk, cyber insurance, prevention, self-protection, self-insurance, coherent risk measures, Stackelberg game

In recent years, as the demand for cyber insurance has increased tremendously, cyber insurance markets around the world have been growing and the range of available cyber policies has been continuously expanding. Many cyber experts agree that pure risk transfer cannot be an optimal cyber risk management solution, and the beneficial potential of including pre-incident and post-incident services into cyber policies is being recognized by insurers and prospective insurance buyers alike (see e.g. [2]).

This talk addresses the question of pricing such services optimally from the insurer's viewpoint, i.e. under which conditions it makes economic sense for a profit-maximizing, risk-averse insurer to share the cost burden of providing such services.

The interaction between buyer and insurer is modelled as a Stackelberg game, where both parties use distortion risk measures with concave distortion functions, yielding a similar problem for the insurance buyer as considered in [1].

After explaining how the notions of pre-incident and post-incident services map to the concepts of self-protection and self-insurance, we detail how in the considered set-up, in the univariate case the insurer will always shift the full cost of self-protection services to the insured. However, this does not generally hold for the pricing of self-insurance services or when taking a multivariate viewpoint, in which case it can be optimal (and even mandatory to find an acceptable contract for both parties) to share the cost of risk reduction service between insurer and policyholder(s).

References

[1] Sarah Bensalem, Nicolás Hernández Santibáñez, and Nabil Kazi-Tani (2020). Prevention efforts, insurance demand and price incentives under coherent risk measures. *Insurance: Mathematics and Economics*, 93(3), 369–386.

[2] Munich RE (2021, March 11), "*Cyber insurance: Risks and trends 2021.*" Retrieved from `https://www.munichre.com/topics-online/en/digitalisation/cyber/cyber-insurance-risks-and-trends-2021.html`, last accessed 14 March 2022.

A model for multifactorial genetic disorders to quantify the impact of polygenic risk scores on life insurance: a simulation based model using heart attack as a case study

22 Aug
PS2

Jinbo Zhao[1], **Michael Salter-Townshend**[2] **and Adrian O'Hagan**[3]

[1]*University College Dublin, Ireland, jinbo.zhao@ucdconnect.ie*
[2]*University College Dublin, Ireland, michael.salter-townshend@ucd.ie*
[3]*University College Dublin, Ireland, adrian.ohagan@ucd.ie*

Keywords: polygenic risk scores, life insurance, premiums, adverse selection

With the evolution of genetic research, our understanding of the genetic architectures of diseases has been enhanced, as well as our ability to predict disease risk. The accompanying progress of direct-to-consumer genetic test services makes it easier for the general public to understand or even predict their own risk of diseases of interest. PRS is one such genetic test result that can add a person's understanding of their disease risk. As the individual-level genetic profile, PRS has the ability to identify high-risk groups when used independently or jointly with other risk factors, and the potential to inform early interventions, which may impact future morbidity and mortality. Although at present PRS can only explain a small proportion of variability in disease risk, the impact of PRS on the foundations of the insurance industry will become increasingly evident as the technology evolves. This study gives a brief picture of what PRS is, what PRS means for insurers and the ways in which PRS can influence customers' insurance choices. Using heart attack as the case study, a simulation-based model is designed to quantify the impact of PRS on insurers under different levels of risk prediction performance. This study employs the liability threshold model (LTM) to simulate the disease liability and PRS, which connects the hidden disease liability of parents with the overall disease liability and PRS of offspring, and links disease prevalence in consecutive age groups with the disease liability for both parents and offspring. The age-specific prevalence rates used in LTM are derived from one-year transition intensities between the healthy state and the heart attack state in a 4-state Markov model, whose intensities are calculated based on data from epidemiology studies. A series of validations are conducted on our simulated data sets. Validation results show that our simulated data can mimic the real data efficiently. For example, we compare the proportion of heart attack events in each PRS strata for different sets of PRS with various risk prediction powers, confirming that even the set of PRS with lowest accuracy is useful in risk prediction. The risk stratification ability between family history and PRS is compared, and we find that family history is a patchy source of information in separating the high risk and low risk groups compared to PRS. The premiums for different PRS risk groups are calculated and compared to the overall premiums when PRS risk information is not available. The extent of the percentage decrease in premium for the low-risk group and the extent of the increase for the high-risk group are calculated for different settings. Adverse selection due to PRS is measured at various insurance market sizes, using different combinations of the proportion of PRS results available and the proportion of insurance purchasing controlling for market size. The accuracy of PRS and the proportion of PRS results available are the main parameters influencing the severity of adverse selection. Our study provides a framework to examine the impact of PRS, which allows extension to PRS with any accuracy or purchasing behaviour of interest.

A hybrid variable annuity contract embedded with living and death benefit riders

22 Aug
PS2

Jennifer Alonso-García[1], **Samuel Thirurajah**[2] **and Jonathan Ziveyi**[3]

[1]*Universite Libre De Bruxelles, Belgium, CEPAR, Netspar, jennifer.alonso.garcia@ulb.be*
[2]*KPMG Sydney, Australia, harrith97@hotmail.com*
[3]*School of Risk and Actuarial Studies, CEPAR, UNSW Sydney, j.ziveyi@unsw.edu.au*

Keywords: retirement income products, variable annuities, guaranteed minimum accumulation benefit (GMAB), guaranteed minimum death benefit (GMDB), taxation

This paper considers a hybrid variable annuity (VA) contract embedded with guaranteed minimum accumulation benefit (GMAB) and guaranteed minimum death benefit (GMDB) riders where the policyholder has the option to surrender anytime prior to maturity. The contract promises the return of the premium paid by the policyholder, or a higher rolled-up value, at the end of the investment period or upon policyholder's death. The product design is structured around a two-account setting where management fees and related costs associated with being invested in the underlying mutual fund are deducted from the investment account. Guarantee fees for financing the GMAB and GMDB riders are deducted from a separate cash account both of which are owned by the policyholder from inception of the contract. This setting general enough to accommodate various fee structures proposed in literature and reflects typical product features for some VA contracts traded in Australia, rather than a single investment account setting often specified in existing literature. We have incorporated stochastic mortality in the valuation framework by adopting the Renshaw and Haberman model [3] calibrated to the Australian mortality data from the Human Mortality Database to facilitate numerical illustrations. Our setting naturally leads to a unique valuation framework which we tackle utilising the method of lines algorithm [1, 2]. We analyse the interplay of management and guarantee fees under the two-account setting and how they influence policyholder surrender behaviour. Retirement income securities are tax free beyond preservation age of 60 in Australia, as such, we analyse policyholder behavior prior and past this preservation age.

References

[1] Meyer, G.H. and Van der Hoek, J. (1997). The valuation of American options with the method of lines. *Advances in Futures and Options Research* 9, 265–286.

[2] Kang, B. and Ziveyi, J. (2018). Optimal surrender of guaranteed minimum maturity benefits under stochastic volatility and interest rates. *Insurance: Mathematics and Economics* 79, 43–56.

[3] Renshaw, A. E. and Haberman, S. (2003). Lee–Carter mortality forecasting with age-specific enhancement. *Insurance: Mathematics and Economics* 33(2), 255–272.

Martingale approach to derive Lundberg-type inequalities

22 Aug
PS1

Jonas Šiaulys

Vilnius University, Lithuania, jonas.siaulys@mif.vu.lt

Keywords: exponential estimate, supremum of sums, tail probability, risk model, inhomogeneity, ruin probability, Lundberg's inequality

Let $\{\xi_1, \xi_2, \ldots\}$ be a sequence of independent and real-valued random variables (r.v.s). Let $\mathcal{S}_0 = 0$, $\mathcal{S}_n = \xi_1 + \xi_2 + \ldots + \xi_n$ for all $n \in \mathbb{N}$ and $\mathcal{M}_\infty = \sup\{\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2, \ldots\}$. In 1997, Sgibnev [1] obtained the upper bound for moment $\mathbb{E}\,\varphi(\mathcal{M}_\infty)$ in the case of independent and identically distributed random variables and a non-decreasing submultiplicative function $\varphi : [0, \infty) \to [0, \infty)$.

If $\mathbb{E}\xi_1 < 0$ and $\mathbb{E}\,\mathrm{e}^{h\xi_1} < \infty$ for some positive h, then the Sgibnev result for the exponential function φ implies that

$$\mathbb{P}\big(\mathcal{M}_\infty > x\big) \leqslant c_1 \mathrm{e}^{-c_2 x},\ x \geqslant 0,$$

with some positive constants c_1 and c_2.

If a sequence $\{\xi_1, \xi_2, \ldots\}$ consists of independent but possibly differently distributed random variables, then the similar estimate also can be derived. In [2], we derive the similar inequality under the following conditions

$$(i)\quad \limsup_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \mathbb{E}\big(\mathrm{e}^{h\xi_k}\big) < \infty \ \text{ for some } \ h > 0,$$

$$(ii)\quad \limsup_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \mathbb{E}\,\xi_k < 0,$$

$$(iii)\quad \limsup_{\substack{u\to\infty \\ n\to\infty}} \frac{1}{n} \sum_{k=1}^{n} \mathbb{E}\left(|\xi_k| \mathbb{I}_{\{\xi_k \leqslant -u\}}\right) = 0,$$

For the proof we use the martingale approach together with the fundamental Wald's identity. As an application we derive a few Lundberg-type inequalities for the ultimate time ruin probability of the inhomogeneous renewal risk model.

References

[1] Sgibnev, M.S. (1997). Submultiplicative moments of the supremum of a random walk with negative drift. *Statistics and Probabability Letters* 32, 377–383.

[2] Kuras, T., Sprindys, J. and Šiaulys, J. (2020). Martingale approach to derive Lundberg-type inequalities. *Mathematics* 8, 1472.

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