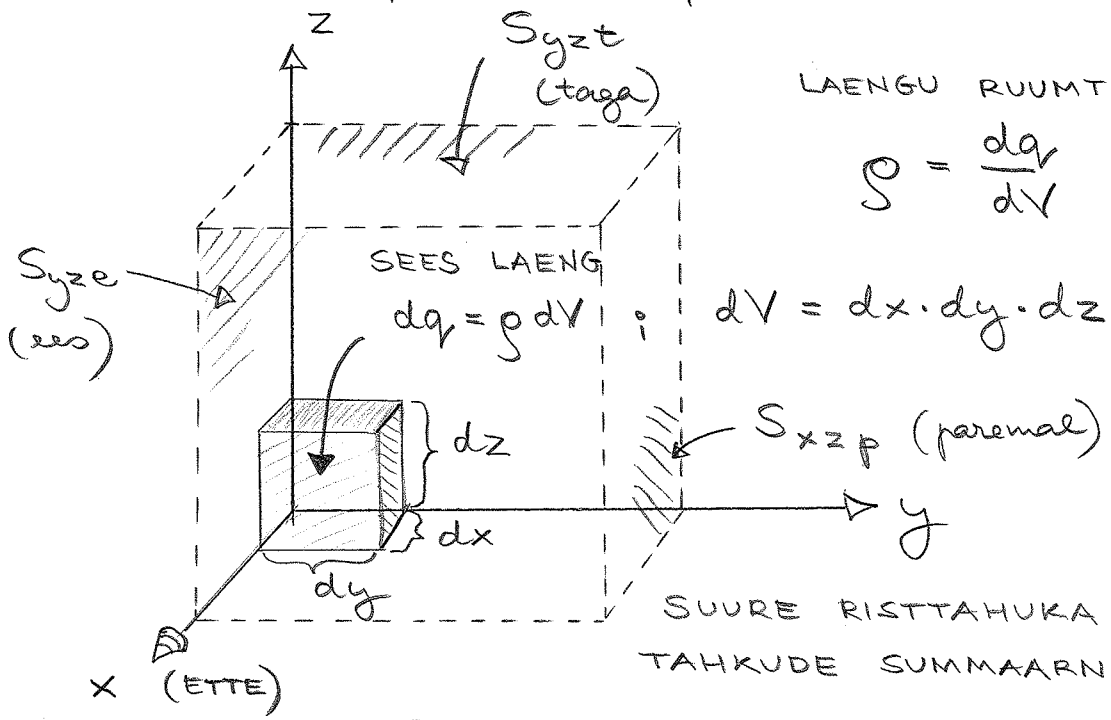
PIDEVUS, POISSON, LAPLACE



LAENGU RUUMTIHEDUS

$$\rho = \frac{dq}{dV}$$

$$dV = dx \cdot dy \cdot dz$$

SUURE RISTTAHUKA KÕIGI TAHKUDE SUMMAARNE PINDALA S'

RISTTAHUKA SEES LAENG

$$Q = \int \rho \, dV$$

RISTTAHUKA RUUMALA $\longrightarrow V'$

SEE Q "VÄHENEB", KUI PINDA S' LÄBIB VOOL (SEEST VÄLJAPOOLE):

$$-\frac{d}{dt} Q = \oint_{S'} J_n \, dS = \dots$$

VOOLUTUGEVUS ON VOOLUTIHEDUSE $\vec{J}$ VOOG

J_x GRADIENT

$$\dots = \int_{S_{yze}} \left(J_x + \frac{\partial J_x}{\partial x} dx \right) dy\,dz - \dots$$

J_x MUUTUS PIKKUSEL dx

VOOL EEST VÄLJA

$$\dots - \int_{S_{yzt}} J_x \, dy\,dz + \int_{S_{xzp}} \left(J_y + \frac{\partial J_y}{\partial y} dy \right) \dots$$

VOOL TAGANT SISSE

VOOL PAREMALT VÄLJA

$$\dots \overset{\text{jne}}{=} \int_{S_{yze}\,S_{yzt}} \frac{\partial J_x}{\partial x} dx\,dy\,dz + \int_{S_{xzp}\,S_{xzv}} \frac{\partial J_y}{\partial y} dy\,dx\,dz + \int_{S_{xy\ddot{u}}\,S_{xya}} \dots$$

VOOL LÄBI TAHKUDE PAARI PIKI X-TELGE TAGANT ETTE ("VÄLJA" − "SISSE")

PIKI Y-TELGE PAREMALE

PIKI Z-TELGE ÜLES

$$= \oint_{S'} \left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} \right) dx\,dy\,dz$$

SEEGA

VOOG ——— ka dV!

$$-\frac{d}{dt}\int_{V'} \rho \underbrace{dx\,dy\,dz}_{dV} = \oint_{S'} \left(\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z}\right) dx\,dy\,dz$$

V' ON PIIRATUD PINNAGA S'

VÕTAME $\frac{d}{dV}$ MÕLEMAST POOLEST

$$\frac{d}{dV}\int_{V'}\left(-\frac{d}{dt}\rho\right) dV = \frac{d}{dV}\oint_{S'} \ldots$$

$\frac{d}{dV}$ ja $\int \ldots dV$ KOMPENSEERUVAD

VOO RUUMTIHEDUS ON DIVERGENTS

$$\operatorname{div}\vec{j} = \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z}$$

$$\Rightarrow \boxed{-\frac{\partial \rho}{\partial t} = \operatorname{div}\vec{j} = \nabla\cdot\vec{j}}$$

NABLA

PIDEVUSE VÕRRAND

MEENUTAME ELEKTROSTAATIKAT...

$$\vec{E} = -\operatorname{grad}\varphi = -\nabla\varphi = \left(\underbrace{-\frac{\partial\varphi}{\partial x}}_{E_x}, \underbrace{-\frac{\partial\varphi}{\partial y}}_{E_y}, \underbrace{-\frac{\partial\varphi}{\partial z}}_{E_z}\right)$$

NABLA

$$\vec{D} = \varepsilon_0 \varepsilon \vec{E}\ ;$$

$$\operatorname{div}\vec{D} = \rho \quad \text{(GAUSS DIFERENTSIAALKUJUL e. VÕRRAND POISSON'i)}$$

$$\nabla\cdot\vec{D} = \nabla\cdot\varepsilon_0\varepsilon(-\nabla\varphi) = \rho$$

↑ SKALAARSELT!

$$\underbrace{\nabla\cdot\nabla}_{\nabla^2 = \Delta}\varphi = -\frac{\rho}{\varepsilon_0\varepsilon}$$

$$\boxed{\Delta\varphi = -\frac{\rho}{\varepsilon_0\varepsilon}}$$

LAPLACE'i OPERAATOR (LABLA)

VABU LAENGUKANDJAID MITTE SISALDAVAS AINES (VÕI VAAKUMIS) $\rho = 0$

JA

KOORDINAATIDES:

$$\frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial^2}{\partial z^2}\varphi = 0$$

$$\Delta\varphi = 0$$

LAPLACE' I VÕRRAND