**Comparison between the translation and rotation:**

| Translation | Rotation |
|---|---|
| The basic quantity is **coordinate** $x$ – the distance of a moving body from the reference body along some choosen axis. | The basic quantity is **coordinate angle** $\theta$ – the angle with respect to the reference position of the rotating body (reference line). |
| The **initial value** of the coordinate $x_0 = x\ (t = 0)$ | The **initial value** of the coordinate angle $\theta_0 = \theta\ (t = 0)$ |
| The **displacement** $s = x - x_0 = \Delta x$ reached during the time interval $\Delta t = t' - t_0$ ($\Delta t = t'$ if $t_0 = 0$) | The **angular displacement** $\varphi = \theta - \theta_0 = \Delta\theta$ reached during the time interval $\Delta t = t' - t_0$ ($\Delta t = t'$ if $t_0 = 0$) |
| The **velocity** in case of the uniform translation: $v = \frac{\Delta x}{\Delta t} = \frac{s}{t}$ <br> SI unit: meter per second (1 m/s) | The **angular velocity** in case of the uniform rotation: $\omega = \frac{\Delta\theta}{\Delta t} = \frac{\varphi}{t}$ <br> SI unit: radian per second (1 rad/s or 1 $s^{-1}$) |
| The equation of motion in case of the uniform translation: <br> $x = x_0 + v\,t$ | The equation of motion in case of the uniform rotation: <br> $\theta = \theta_0 + \omega\,t$ |
| The **average velocity** of the non-uniform translation: $v_{avg} = \frac{\Delta x}{\Delta t}$ | The **average angular velocity** of the non-uniform rotation: $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ |
| The **instantaneous velocity**: $v(t) = \lim \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$ , $\Delta t \to 0$ | The **instantaneous angular velocity** $\omega(t) = \lim \frac{\Delta\theta}{\Delta t} = \frac{\mathrm{d}\theta}{\mathrm{d}t}$ , $\Delta t \to 0$ |
| The constant **acceleration**: $a = \frac{v - v_0}{t}$ (SI unit 1 m/$s^2$) <br> Dependence of the velocity $v$ on time $t$: $v = v_0 + a\,t$ | The constant **angular acceleration**: $\alpha = \frac{\omega - \omega_0}{t}$ (SI unit 1 rad/$s^2$) <br> Dependence of the angular velocity $\omega$ on time $t$: $\omega = \omega_0 + \alpha\,t$ |
| The **instantaneous** value of acceleration: $a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ | The **instantaneous** value of angular acceleration: $\alpha(t) = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$ |
| The equation of motion in case of the **constant** acceleration: <br> $x = x_0 + v_0 t + \frac{a\,t^2}{2}$ | The equation of motion in case of the **constant** angular acceleration: <br> $\theta = \theta_0 + \omega_0 t + \frac{\alpha\,t^2}{2}$ |
| The case of **unknown** time interval: $v^2 - v_0^2 = 2\,a\,s$ | The case of **unknown** time interval: $\omega^2 - \omega_0^2 = 2\,\alpha\,\varphi$ |
| The velocity of change of acceleration: $b = \frac{a - a_0}{t}$ (m/$s^3$) <br> Dependence of the acceleration $a$ on time $t$: $a = a_0 + b\,t$ | The velocity of change of angular acceleration: $\beta = \frac{\alpha - \alpha_0}{t}$ ($s^{-3}$) <br> Dependence of the angular acceleration $\alpha$ on time $t$: $\alpha = \alpha_0 + \beta\,t$ |
| The equation of motion in case of the **constant** parameter $b$: <br> $x = x_0 + v_0\,t + a_0\,t^2/2 + b\,t^3/6$ | The equation of motion in case of the **constant** parameter $\beta$: <br> $\theta = \theta_0 + \omega_0\,t + \alpha_0\,t^2/2 + \beta\,t^3/6$ |

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| The intensity of the interaction is described by **force** $\mathbf{F}$<br>SI unit newton: $1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m/s^2}$. | The action of a force on the rotation of the body is described by **torque** $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ (vector product). SI unit newton-meter: $1\ \mathrm{N} \cdot \mathrm{m}$ |
| The property of the body to maintain its state of the translational motion (the property of inertia) is described by the (inertial) **mass** $\boldsymbol{m}$ (SI unit 1 kg) | The property of the body to maintain its state of the rotational motion is described by the **rotational inertia** $\boldsymbol{I}$ (SI unit $1\ \mathrm{kg} \cdot \mathrm{m^2}$). The single particle with the mass $m$ rotating at the distance $r$ (radius) from the rotational axis has the rotational inertia $I = m\, r^2$. |
| The **Newton**'s **1**-st law: If $\mathbf{F}_{\mathrm{net}} = 0$, then $\boldsymbol{a} = 0$ and $\mathbf{v} = \mathrm{const}$.<br>The body is at rest or in uniform motion along the straight line. | The **Newton**'s **1**-st law: If $\boldsymbol{\tau}_{net} = 0$, then $\boldsymbol{\alpha} = 0$ and $\boldsymbol{\omega} = \mathrm{const}$.<br>The body is at rest or performs uniform rotation. |
| The **Newton**'s **2**-nd law: If $\mathbf{F}_{\mathrm{net}} \neq 0$, then $a = \frac{1}{m} F_{net}$, $\mathbf{F}_{\mathrm{net}} = m\, \boldsymbol{a}$<br>The body moves translationally with an acceleration which is proportional to the net force $\mathbf{F}_{\mathrm{net}}$. | The **Newton**'s **2**-nd law: If $\boldsymbol{\tau}_{\mathrm{net}} \neq 0$, then $\alpha = \frac{1}{I} \tau_{net}$, $\boldsymbol{\tau}_{\mathrm{net}} = I\, \boldsymbol{\alpha}$<br>The body moves rotationally with an angular acceleration which is proportional to the net torque $\boldsymbol{\tau}_{\mathrm{net}}$. |
| The **Newton**'s **3**-rd law: $\mathbf{F}_{12} = -\mathbf{F}_{21}$.<br>The force $\mathbf{F}_{12}$ exerted by one body (1) onto the other one (2) is equal by magnitude to the force $\mathbf{F}_{21}$ exerted by second body (2) on the first one (1), whereas these forces have opposite directions. | The **Newton**'s **3**-rd law: $\boldsymbol{\tau}_{12} = -\boldsymbol{\tau}_{21}$.<br>The torque $\boldsymbol{\tau}_{12}$ exerted by one body (1) onto the other one (2) is equal by magnitude to the torque $\boldsymbol{\tau}_{21}$ exerted by second body (2) on the first one (1), whereas these torques have opposite directions. |
| The linear momentum $\mathbf{p} = m\, \mathbf{v}$ shows us the ability of the translationally moving body to bring other bodies into motion (SI unit $1\ \mathrm{kg} \cdot \mathrm{m/s}$). | The angular momentum $\mathbf{L} = I\, \boldsymbol{\omega}$ shows us the ability of the rotationally moving body to bring other bodies into motion (SI unit $1\ \mathrm{kg} \cdot \mathrm{m^2/s}$). The single particle with the mass $m$ rotating at the distance $r$ (radius) from the rotational axis with the velocity $v$ has the angular momentum $L = m\, v\, r$. |
| The net linear momentum of the isolated system is conserved:<br>$\Sigma\, \mathbf{p}_i = \mathrm{const}$ | The net angular momentum of the isolated system is conserved:<br>$\Sigma\, \mathbf{L}_i = \mathrm{const}$ |
| The fundamental equation of the translational motion (the **Newton**'s second law): $\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$.<br>The force $\mathbf{F}$ causes the change of the linear momentum $\mathbf{p}$. | The fundamental equation of the rotational motion (the **Newton**'s second law): $\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$.<br>The torque $\boldsymbol{\tau}$ causes the change of the angular momentum $\mathbf{L}$. |
| **Work** in case of translation: $W = \mathbf{F} \cdot \mathbf{s}$ (scalar product) | **Work** in case of rotation: $W = \boldsymbol{\tau} \cdot \boldsymbol{\varphi}$ (scalar product) |
| **Kinetic energy** in case of translation: $E_{k,tr} = \frac{m\, v^2}{2}$ | **Kinetic energy** in case of rotation: $E_{k,rot} = \frac{I\, \omega^2}{2}$ |