HARMONIC OSCILLATIONS

Newton's 2nd law: $F_{rest} = m\,a$

F_{rest} – restoring force.

$F_{rest} = F_{el} = -k\,x$ (Hooke's law),

where k is the force coefficient.

So we have: $-k\,x = m\,a$ and $m\,a + k\,x = 0.$

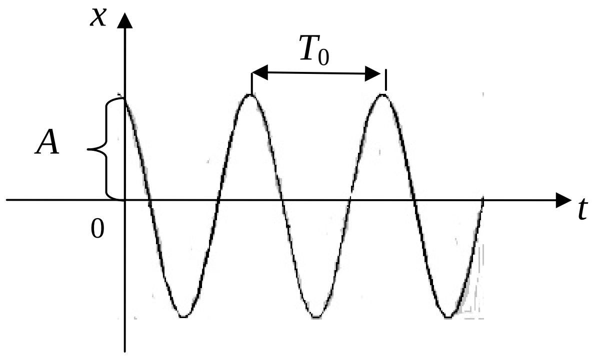
Because the acceleration a is the second derivative of the coordinate x:

$a = \frac{d}{dt}\frac{d}{dt}x = \ddot{x}$, the **differential equation** of the oscillations has the form:

$\ddot{x} + \frac{k}{m}x = 0$ and its solution is:

$x = A\cos\omega_0 t,$ where the angular frequency is $\omega_0 = \sqrt{\frac{k}{m}}$ and the period

$T_0 = 2\pi\sqrt{\frac{m}{k}}.$



DAMPED OSCILLATIONS

$F_{rest} + F_{drag} = m\,a$

$F_{drag} = -\,bv$, where

b is the drag coefficient.

So we have: $-k\,x - b\,v = m\,a$

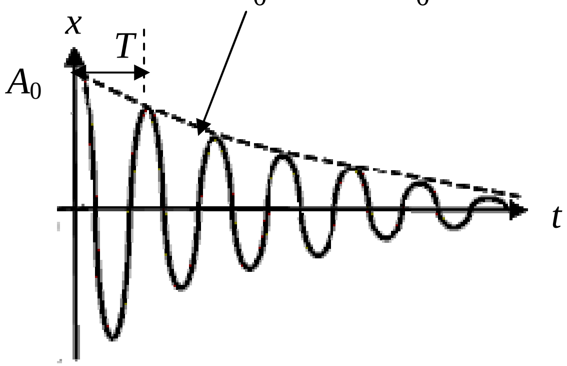
or $m\,a + b\,v + k\,x = 0.$ Dividing by mass m

we get $a + \frac{b}{m}v + \frac{k}{m}x = 0$, the **differential equation**: $\ddot{x} + 2\frac{b}{2m}\dot{x} + \frac{k}{m} = 0$

and its solution: $x = A_0\,e^{-\beta t}\cos\omega t$

where $\beta = \frac{b}{2m}$; $\omega = \sqrt{\omega_0^2 - \beta^2}$

$A = A_0\,e^{-\beta t} = A_0\,e^{-t/\tau}$



$\tau = \frac{1}{\beta}$ - time constant

FORCED OSCILLATIONS

$F_{rest} + F_{drag} + F_{out} = m\,a$

$ma + bv + kx = F_{out}$

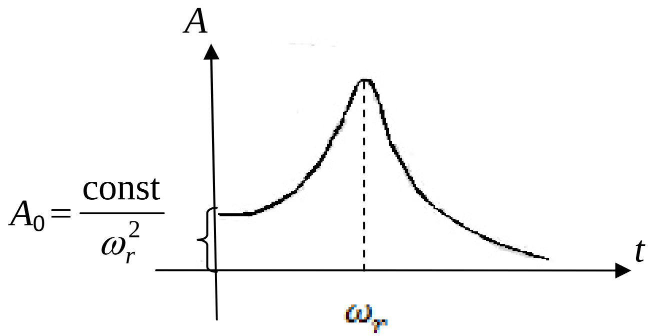
$a + \frac{b}{m}v + \frac{k}{m}x = \frac{F_m}{m}\cos\omega t$

$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = a_m\cos\omega t$

The solution of this **differential equation** is: $x = A(\omega)\cos(\omega t + \varphi)$

where the phase angle $\varphi = \varphi(\beta, \omega_0)$ and the amplitude A depend on the frequency ω of the periodic force F_{out} acting from outside: $A = A(\omega)$.

$A(\omega) = \frac{\text{const}}{\left|\omega_r^2 - \omega^2 - i\beta\omega\right|}$



the resonance frequency $\omega_r = \sqrt{\omega_0^2 - 2\beta^2}$