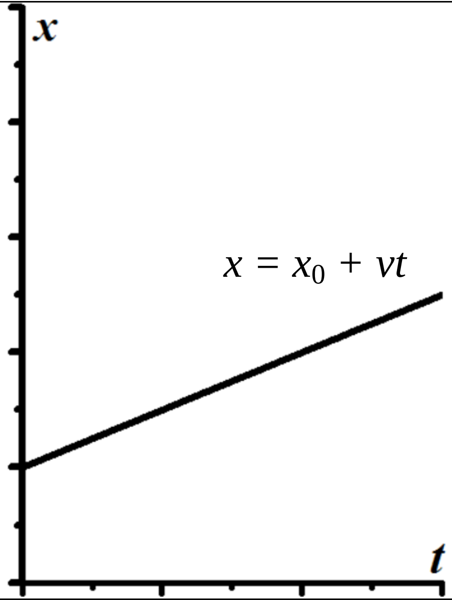
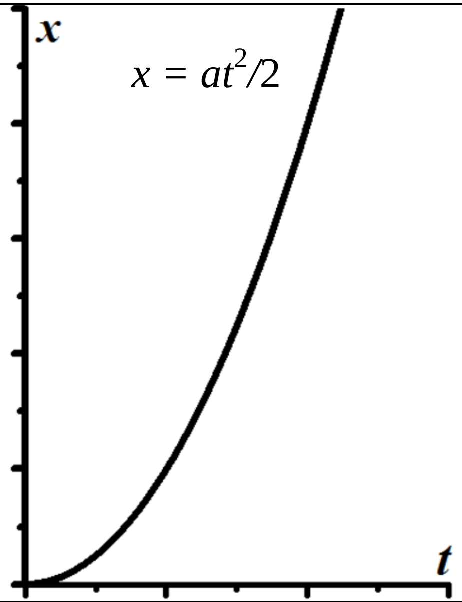
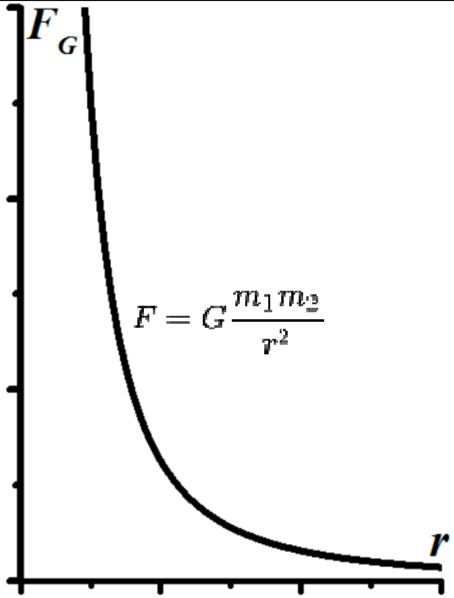
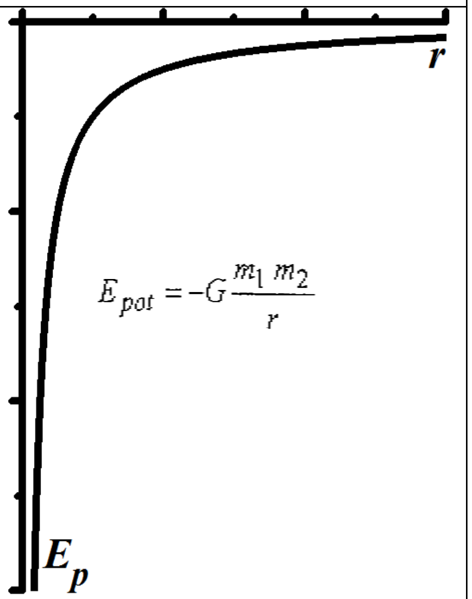
Functions in physics are either power functions or exponential functions:

Power functions (PF): Polynome $y = ax^n + bx^{n-1} + c\,x^{n-2} + ..$ is a sum of various power functions.				
Special cases of PF:	Linear function	Square function	Inverse square function	Inverse value function
Mathematical form:	$y = ax$ ($n = 1$)	$y = ax^2$ ($n = 2$)	$y = ax^{-2} = \frac{a}{x^2}$ ($n = -2$)	$y = ax^{-1} = \frac{a}{x}$ ($n = -1$)
Examples from physics:	a) <u>Equation of motion</u> at the uniform motion: $x = v\,t$ The coordinate of the body x depends in linear way on time t. The speed v is a constant. b) <u>The linear momentum</u> of the body p depends in linear way on the speed v: $p = m\,v$. The mass of the body m is a constant. c) <u>Capacitor</u>**:** The electric charge q stored in the capacitor depends in linear way on the voltage U: $q = C\,U$. The capacitance C is a constant.	a) <u>Displacement</u> at the non-uniform motion s depends on the time t (assuming $x_0 = 0$, $v_0 = 0$): $s = \frac{at^2}{2}$. b) <u>Kinetic energy</u> of the body E_k depends on the speed v: $E_k = \frac{mv^2}{2}$. c) <u>Energy of the electric field</u> E_e depends on the voltage U: $E_e = \frac{CU^2}{2}$.	a) <u>The gravitational force:</u> $F_G = G\frac{m_1\,m_2}{r^2}$, where m_1 and m_2 are the masses of interacting bodies and r is the distance between the centres of mass of these bodies. b) <u>Electric field strength</u> E caused by the point charge q at the distance r from this charge: $E = k\frac{q}{r^2}$ k – Coulomb constant.	a) <u>The potential energy</u> of the gravitational force: $E_{pot} = -G\frac{m_1\,m_2}{r}$ Abbreviations are the same (←), G is the Newton gravitation constant. b) <u>Potential</u> V or φ caused by the point charge q at the distance r from this charge: $\varphi = k\frac{q}{r}$ k – Coulomb constant.
Graphical represen-tation:				

The derivative of the function is the value of the speed of change of the function at the given time:	The speed of change of the linear function is constant: $v_t = \frac{\mathrm{d}}{\mathrm{d}t}x$ = const	The speed of change of the power function can be described by a function that has an exponent that is by one number smaller than the initial function. For instance, the speed of change of square function can be described as a linear function. $v_t = \frac{\mathrm{d}}{\mathrm{d}t}x = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{at^2}{2}\right) = \frac{a}{2}2t = at$ (speed at the moment of time t or the final speed)

The **derivative** or the **integral** of the function:	Linear function: example (b) upwards, derivative is constant: $\frac{\mathrm{d}}{\mathrm{d}v}p = \frac{\mathrm{d}}{\mathrm{d}v}(mv) = m$ = const	Square function: example (a) Displacement = average speed * time: $s = \frac{0+at}{2} * t = \frac{at^2}{2}$ (initial speed = 0, final speed $v = a\,t$)	Inverse square function: example (b) $\int_1^2 E\,\mathrm{d}r = \int_1^2 k\frac{q}{r^2}\mathrm{d}r = k\frac{q}{r_1} - k\frac{q}{r_2} = U_{12}$ The integral of the electric field strength is the potential difference between the two field points or the voltage.	Inverse value function: example (b) $\frac{\mathrm{d}}{\mathrm{d}r}\varphi = \frac{\mathrm{d}}{\mathrm{d}r}\left(k\frac{q}{r}\right) = -k\frac{q}{r^2} = -E$. The derivative of the potential φ = – field strength E. If we move along the field direction, then the potential **decreases** (sign „minus“).

The **property** of the nature or the model describing it, which causes exactly such kind of functional dependence between physical quantities.	The integral of the linear function is the square function. The formula $s = \frac{0+at}{2} * t = \frac{at^2}{2}$ represents the graphical integration of the linear function $v = a\,t$. In the case of the non-linear functions the changes are observed in the interval of the argument values wide enough to establish the non-linearity (the graph of the function cannot be depicted as a line segment).	The inverse square function is the dependence of a force characteristic of the field from the distance between the body exerting the force and the point of interest. In essence, this is a law of geometry: a statement that the area can be calculated as a square function of a linear dimension. Thus, if the distance of the point of interest from the point source undergoes n-fold increase, then the area of the imaginary sphere surrounding such point source undergoes n^2-fold increase. This is in turn accompanied by the n^2-fold decrease of the probability of a field particle to reach exactly this point of interest. Field strength (FS) = const $\frac{q\,\text{(charge)}}{A\,\text{(area)}}$	The inverse value function is describing the dependence of an energy characteristic (that is, an integral of the force characteristic) of an FS = const q/A type of field (here FS stands for field strength) from the distance between the body exerting the force and the point of interest. **NB!** The prerequisite for the development of the FS = const q/A type of field is the endless life-span of the interaction-mediating particle (field particle). It means that the particle does not undergo a spontaneous decomposition.

Exponential functions (EF): the common mathematical form is $y = a\, e^{\pm bx} = a \exp(\pm bx)$

Definition: $e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ..$, where the factorial $n!$ of the number n is defined as the product: $n! = 1 \cdot 2 \cdot 3 \cdot (n-1) \cdot n$.

The Euler number $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + .. = 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + ..$ = 1 + 1 + 0.5 + 0.167 + 0.042 + 0.008 + …= **2.7183..**

Such kind of sum is called **converging series** in mathematics. If $x << 1$, then we get a very helpful formula $e^x \approx 1 + x$.

For instance $e^{0,1} \approx 1 + \frac{0.1}{1} = 1.1$. The exact value is $e^{0.1} = 1 + \frac{0.1}{1} + \frac{0.01}{2} + \frac{0.001}{6} + \frac{0.0001}{24} + \ldots$ = 1 + 0.1 + 0.005 + 0.000167 + … = 1.105171.

The difference can be neglected.

$e^2 = 1 + \frac{2^1}{1} + \frac{2^2}{2} + \frac{2^3}{6} + \frac{2^4}{24} + \frac{2^5}{120} + ..$ = 1 + 2 + 2 + 1.333 + 0.667 + 0.267 + .. ≈ 7.4.

$e^3 = 1 + \frac{3^1}{1} + \frac{3^2}{2} + \frac{3^3}{6} + \frac{3^4}{24} + \frac{3^5}{120} + ..$ = 1 + 3 + 4.5 + 4.5 + 3.375 + 2.025 + .. ≈ 20.

The simple exponential function can be presented in the form of the **power series**: $\exp x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots$

The **cosine** function $\cos x$ is an **even** function [$\cos(-x) = \cos x$] and it can be presented in the form of the power series, consisting of the **even** terms of the series of the exponential function, but in such a way that the signs in front of the terms are varying (+ and –):

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

The **sine** function $\sin x$ is an **odd** function [$\sin(-x) = -\sin x$] and it can be presented in the form of the power series, consisting of the **odd** terms of the series of the exponential function, but in such a way that the signs in front of the terms are varying (+ and –):

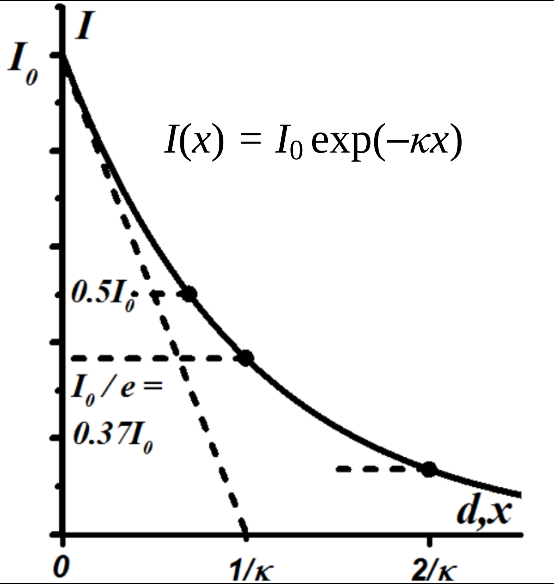
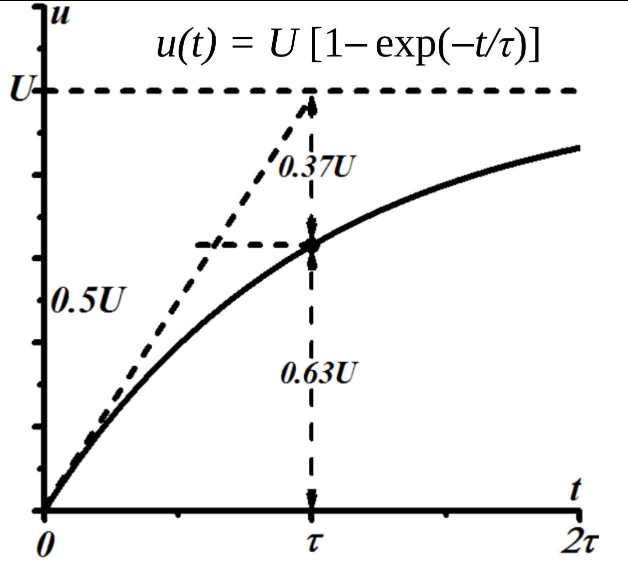
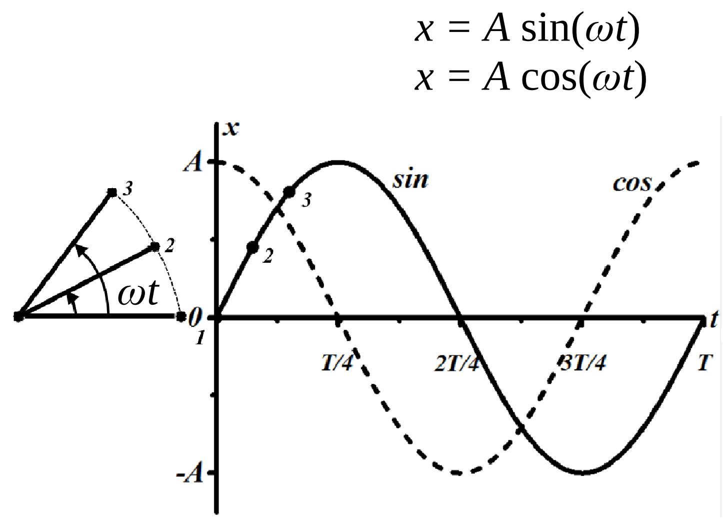
$$\sin x = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots ,$$

Now we have

$$\exp(ix) = 1 + \frac{(ix)^1}{1!} + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + .. = 1 + i\frac{x^1}{1!} + i^2\frac{x^2}{2!} + i^3\frac{x^3}{3!} + i^4\frac{x^4}{4!} + i^5\frac{x^5}{5!} + .. = 1 + i\frac{x^1}{1!} - \frac{x^2}{2!} - i\frac{x^3}{3!} + \frac{x^4}{4!} - i\frac{x^5}{5!} + .. = \cos x + i \sin x.$$

So we get the **Euler formula** $\exp(ix) = \cos x + i \sin x$, which enables us to switch from the trigonometric form of the complex number to the exponential form of the complex number.

According to the Euler formula, the trigonometric sine and cosine functions are the exponential functions, too: $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$ and $\cos x = \frac{e^{ix} + e^{-ix}}{2}$.

Special cases:	Exponential decay	Limited exponential increase	Sine function	Cosine function
Mathematical form:	$y = Ae^{-bx} = \frac{A}{e^{bx}}$	$y = A(1-e^{-bx}) = A - \frac{A}{e^{bx}}$	$y = A\sin x = A\frac{e^{ix}-e^{-ix}}{2i}$	$y = A\cos x = A\frac{e^{ix}+e^{-ix}}{2}$
Examples from physics:	a) Decrease of the amplitude A as a function of time t for the damped oscillations**:** $A(t) = A_0 \exp(-\beta t) = A_0 \exp(-t/\tau)$, where A_0 is the initial amplitude, $\beta = 1/\tau$ is the coefficient of damping and τ is the time constant. b) The law of the radioactive decay: $N(t) = N_0 \exp(-t/\tau)$, where $N(t)$ is the number of the radioactive nuclei at the time t, N_0 – the initial number of the radioactive nuclei and τ – time constant. c) The law of the absorption of the light: $I(x) = I_0 \exp(-\kappa x)$, where I_0 is the intensity of the incident light and $I(x)$ is the intensity of the light after passing the distance x from the surface, κ – absorption coefficient (in the units cm^{-1}).	a) Increase of the voltage $u(t)$ between the plates of a capacitor used as the energy storage, as a function of time t**:** $u(t) = U_m[1-\exp(-t/\tau)]$. U_m is the limit (maximum of the voltage). In fact it is the electromotive force of the source used by the charging of the capacitor. τ is the time constant. b) Increase of the current $i(t)$ through the inductor used for the energy storage, as a function of time t**:** $i(t) = I_m[1-\exp(-t/\tau)]$. $I_m = U_m/R$ is the limit (maximum of the current), where R is the resistance of the circuit used. Time constants: In the case (a) $\tau = RC$, where R is the resistance and C is the capacitance. In the case (b) $\tau = L/R$, where R is the resistance and L is the inductance of the circuit used.	a) Equation of motion at the harmonic oscillations: $x(t) = A\sin\omega t$ or $x(t) = A\cos\omega t$, where the maximum value A of the coordinate x is called amplitude, the angle ωt determining the coordinate of the oscillating body is called phase and the speed of the changing of the phase is called angular frequency ω. b) Alternating current (AC): $i(t) = I_m\sin\omega t$ or $i(t) = I_m\cos\omega t$, where the maximum value I_m of the instantaneous current i serves as the amplitude. The sine function is used in case if we start counting time at the equilibrium state of the pendulum ($x = 0$, when $t = 0$) The cosine function is used in case if we start counting time at the maximally deviant state of the pendulum ($x = A$, when $t = 0$)	
Graphical represen-tation:				

The derivative of the function $v_x = \frac{d}{dx} y$ or $v_t = \frac{d}{dt} y$	$v_t = \frac{d}{dt} A_0 e^{-\frac{t}{\tau}} = -\frac{A_0}{\tau} e^{-\frac{t}{\tau}}$ (case a)	$v_t = \frac{d}{dt}\left[U - U e^{-\frac{t}{\tau}}\right] = \frac{U}{\tau} e^{-\frac{t}{\tau}}$	$v_t = \frac{d}{dt} A\sin\omega t =$ $\omega A \cos \omega t = \omega A \cos(-\omega t) =$ $\omega A \sin[\pi/2 - (-\omega t)] = \omega A \sin(\omega t + \pi/2)$
The derivative of the function as the instantaneous value of the speed of change of the function.	**The derivative** is also exponential function but has a phase shift of $\boldsymbol{\pi}$ radians with respect to the initial function. It means that if the initial function is exponential decay then its derivative is limited exponential increase. If the initial function is limited exponential increase then its derivative is exponential decay. So the graphs of the function and its derivative are mirror images of each other with respect to the argument axis.		**The derivative** is also harmonic (sine or cosine) function but it has a phase shift **+** $\boldsymbol{\pi/2}$ radians with respect to the initial function. It means that all which happens with the function also happens with the derivative but in quarter of period earlier. The speed of change (derivative) leads the function itself.
The **property** of the nature or the model describing it, which causes exactly such kind of functional dependence between physical quantities.	The system possesses an equilibrium state and the **restoring force** (force directed to the equilibrium state) is acting on the system and determining its motion. But the resistive force or **drag force** is also present and its impact is comparable to the impact of the restoring force. As a result the changes are **slow** and the inertial properties of the system are **not** important. The system is asymptotically approaching the equilibrium state and does not exceed it. The oscillations do not occur. **NB!** The exponential dependence occurs when the **change** of the physical quantity studied is proportional to the initial value of this quantity (function). The change is proportional to the remaining amount. Only that can change what has not changed yet. *Only those can die, who are yet alive.*		The system possesses an equilibrium state and the **restoring force** (force directed to the equilibrium state) is acting on the system and determining its motion. The resistive force or **drag force** is also present but its impact is negligible with respect to the restoring force. As a result the changes are **fast** and the inertial properties of the system are becoming **important**. The system does not stop at the equilibrium state. It continues to move inertially and **exceeds** the equilibrium state. The oscillations occur. **NB!** An important parameter is the **ratio** of the quantity describing the inertial properties of the system (mass m in the case of mechanics, inductance L in the case of electromagnetic field) and the quantity describing the drag force (drag coefficient b in the case of mechanics, resistance R in the case of electromagnetic field). The oscillations occur only when this ratio is essentially bigger than the half-period $T_0/2$ of the oscillations. In the case of inductor (see above) the ratio aforementioned is the time constant $\tau = L/R$ of the LR-circuit. So the condition for the occurrence of the oscillations is $\frac{L}{R} > \frac{T_0}{2}$.