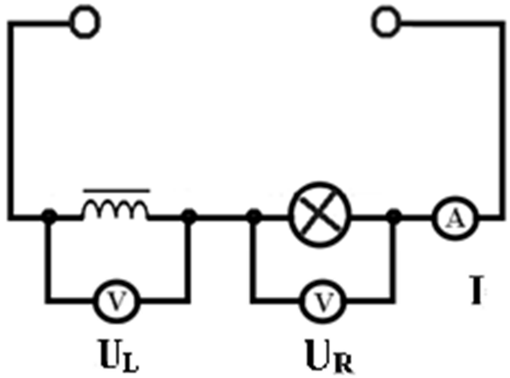
Alternating current experiment (*Physics and Engineering* lecture, the 17th of December 2018)

a) An inductor (DC resistance of 1.5 Ω) and a lamp with nominal power of 100 W connected in **series**:

Network voltage ~ 237 V (RMS value)

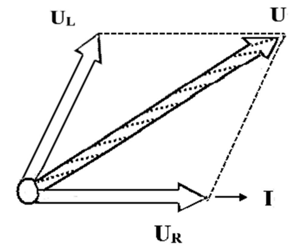


The results of measurements: the **voltage** applied on the inductor U_L = 154 V, the voltage on the lamp U_R = 159 V, the sum of those is equal to 313 V but this sum is **not equal** to the network voltage 237 V. I = 0.37 A.

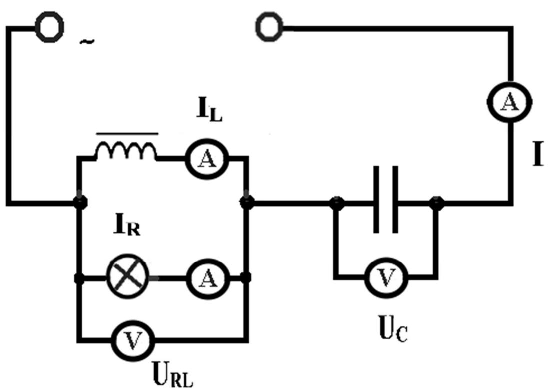
The phasors of the voltages also do **not** follow the Pythagorean theorem:

$\sqrt{154^2 + 159^2} = 221 \neq 237$.

Conclusion: the phasors of voltages possessing lengths $\sqrt{2}$ times U_L and U_R are neither parallel (the case of a DC circuit) nor perpendicular (the case of an ideal inductor). There is some angle between them which is less than the right one.



b) The inductor and the lamp connected in **parallel** and a capacitor (C = 8 μF) connected in **series** with this parallel connection:



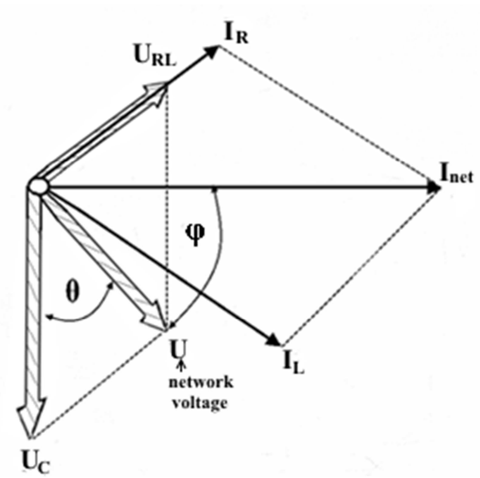
The results of measurements: the **voltage** applied on the parallel connection of the inductor and the lamp U_{RL} = 219 V, the voltage applied on the capacitor U_C = 304 V, the sum of those: 513 V

The currents: in the branch of the inductor I_L = 0.48 A, in the branch of the lamp I_R = 0.41 A, the net current I = 0.75 A. The sum of currents in the branches is **not equal** to the net current measured: 0.48 + 0.41 = 0.89 ≠ 0.75

The phasors of the currents also do **not** follow the Pythagorean theorem:

$\sqrt{0.48^2 + 0.41^2} = 0.63 \neq 0.75$.

Conclusion: the phasors of the currents are neither parallel (the case of a DC circuit) nor perpendicular (the case of an ideal inductor). There is some angle between them which is less than the right one.



Power consumed by the AC circuit: $P = I\,U \cos\varphi$

Additional task: Please show the agreement between the two experiments (*a* and *b*). For this purpose calculate the resistance and the inductance of the inductor (a non-ideal one in our case!) using the data of both experiments. Calculate the power consumed by the inductor and by the lamp in both cases. Calculate the capacitance using experimental data and compare the obtained value with the given one (C = 8 μF).

Some hints:

In the case (*a*) please draw the vertical line from the common starting point of the voltage phasors. This line is now perpendicular to the phasor of the current which is directed rightwards. The phasor of the voltage U_L measured on the inductor can be divided into two components: the first one directed along the vertical line and the second one which is parallel to the horizontal phasor of the current. The vertical component U_{XL} is the voltage on the imagined ideal inductor possessing purely inductive reactance X_L. The horizontal component U_{RL} is the voltage on the resistance R_L of the inductor. This resistance is mostly caused by the eddy currents induced in the iron core and also in the neighbouring metallic bodies because the magnetic circuit (iron core) of the inductor is partially opened (there is some air gap). So we need the angle δ between the vertical and the phasor of U_L, because $U_{XL} = U_L \cos\delta$ and $U_{RL} = U_L \sin\delta$.

Let us denote the angle between the phasors U_L and U_R by α and in the triangle containing the phasors U_L, U_R and U the biggest angle by β (note that $\beta = 180^0 - \alpha$). Now we can find β from the cosine theorem:

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac},$$

where a and c are the neighbouring sides for the angle β, and b is the side located opposite to the angle β. Using the values of voltages U_L = 154 V, U_R = 159 V and U = 237 V we obtain

$$\cos\beta = \frac{154^2 + 159^2 - 237^2}{2 \cdot 154 \cdot 159} = -0.146 \quad \text{and} \quad \beta = 98.4^0.$$

So the angle between the phasors U_L and U_R is equal to 180 – 98,4 = 81,6 degrees and the angle $\delta = 8{,}4^0$. Now it is possible to calculate $U_{XL} = U_L \cos\delta$ and $U_{RL} = U_L \sin\delta$. If we divide those by the value of current then we obtain the inductive reactance X_L and the resistance R_L of the inductor. Next we can use the formula $X_L = \omega L$ and find the inductance L dividing X_L by the European angular frequency $\omega = 2\pi * 50\text{ Hz} = 314\text{ s}^{-1}$.

In the case (*b*) we can calculate at first the angle θ between the phasors U and U_C from the cosine theorem. It is reasonable to use always the values of voltages for such calculations because the currents are measured here with very big uncertainties. Next we can find the phase angle φ between the network voltage U and the net current I: $\varphi = 90^0 - \theta$, because the capacitor has no resistance, the voltage on the capacitor U_C lags behind the net current I and the angle between the phasors of U_C and I is the right angle (90^0). Now we are able to calculate the power P consumed by the whole circuit using the formula $P = I\,U\cos\varphi$. If we subtract from this the power of the lamp $I_R \cdot U_R$, then we obtain the power consumed by the resistance R_L of the inductor. Applying Ohm's law for the capacitor, we can calculate the capacitive reactance and finally the capacitance itself.