

Sumbuvate võnkumiste diferentsiaalvõrrand: $x'' + 2\beta x' + \omega_0^2 x = 0$

Selle lahend:

$$x = A(t) e^{i\omega t}$$

$$A(t) = ? \quad \omega = ?$$

$$x' = A' e^{i\omega t} + A e^{i\omega t} (i\omega)$$

$$\begin{aligned} x'' &= A'' e^{i\omega t} + A' e^{i\omega t} (i\omega) + A' e^{i\omega t} (i\omega) + A e^{i\omega t} (i\omega)(i\omega) = \\ &= A'' e^{i\omega t} + 2i\omega A' e^{i\omega t} - \omega^2 A e^{i\omega t} \end{aligned}$$

paneme algvõrrandisse sisse:

$$A'' e^{i\omega t} + 2i\omega A' e^{i\omega t} - \omega^2 A e^{i\omega t} + 2\beta A' e^{i\omega t} + 2i\omega \beta A e^{i\omega t} + \omega_0^2 A e^{i\omega t} = 0$$

$$A'' + 2i\omega A' - \omega^2 A + 2\beta A' + 2\beta A i\omega + \omega_0^2 A = 0$$

$$A'' + 2\beta A' + (\omega_0^2 - \omega^2) A = 0 \quad \text{reaalosa kohta}$$

$$2i\omega A' + 2\beta A i\omega = 2i\omega (A' + \beta A) = 0 \quad \text{imaginaarosa kohta}$$

triviaalne lahend: $\omega = 0$ mittetriviaalne lahend: $A' + \beta A = 0$

Sellest $A' = -\beta A$ ehk $dA/dt = -\beta A$ ehk $dA/A = -\beta dt$.

Integreerides saame: $\ln A = -\beta t + \text{const}$

Algtingimusest: $\text{const} = \ln A_0$ kus $A_0 = A(t=0)$

Kokku võttes: $\ln A - \ln A_0 = -\beta t$ ehk $\ln (A/A_0) = -\beta t$

Antilogaritmidest saame: $A/A_0 = e^{-\beta t}$ ehk $A = A_0 e^{-\beta t}$

$$A' = -\beta A_0 e^{-\beta t} = -\beta A$$

$$A'' = (-\beta)(-\beta) A_0 e^{-\beta t} = \beta^2 A$$

Paneme need tuletised reaalse võrrandisse $A'' + 2\beta A' + (\omega_0^2 - \omega^2) A = 0$

ja saame, et $\beta^2 A + 2\beta(-\beta A) + (\omega_0^2 - \omega^2) A = 0,$

millest A -ga läbijagamisel saame $\beta^2 - 2\beta^2 + \omega_0^2 - \omega^2 = 0,$

ehk $-\beta^2 + \omega_0^2 - \omega^2 = 0$ ehk $\omega^2 = \omega_0^2 - \beta^2$