Combined Standard Uncertainty

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Combined Standard Uncertainty

- When estimating the standard uncertainty of an **output quantity** then the standard uncertainties of all **input quantities** are taken into account
- The standard uncertainty of the output quantity obtained in this way is called **combined standard uncertainty**
- Is denoted by $u_c(y)$

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Indirect Measurement

- If the result is obtained as an **output quantity** by calculations from one or several **input quantities** then we speak about **indirect measurement**

- Example: Titration
  - Titrant concenration
  - Volume of sample solution taken for titration
  - Volume of titrant consumed for titartion

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Measurement model

- Expression that enables calculating the output quantity value (result value) from the input quantity values

- Titration with 1:1 stoichiometry:

Titrant volume

Titrant concentration

$$C_S = \frac{V_T \cdot C_T}{V_S}$$

Analyte concentration
in sample solution

Volume of sample solution
taken for titration

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Finding u_c

- If the model is:

$$Y = X_1 - X_2 + ... + X_n$$

- then:

$$u_c(y) = \sqrt{u(x_1)^2 + u(x_2)^2 + ... + u(x_n)^2}$$

- All uncertainties have to be converted to **standard uncertainties** before calculations!

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Finding u_c

- If the model is:

$$Y = \frac{X_1 \cdot X_2}{X_3 \cdot X_4}$$

- Then:

$$u_c(y) = y \cdot \sqrt{\left(\frac{u(x_1)}{x_1}\right)^2 + \left(\frac{u(x_2)}{x_2}\right)^2 + \left(\frac{u(x_3)}{x_3}\right)^2 + \left(\frac{u(x_4)}{x_4}\right)^2}$$

$$\frac{u_c(y)}{y} = \sqrt{\left(\frac{u(x_1)}{x_1}\right)^2 + \left(\frac{u(x_2)}{x_2}\right)^2 + \left(\frac{u(x_3)}{x_3}\right)^2 + \left(\frac{u(x_4)}{x_4}\right)^2}$$

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Finding u_c

- The general case:

$$Y = F(X_1, X_2, \ldots, X_n)$$

- then:

$$u_c(y) = \sqrt{\left[\frac{\partial Y}{\partial X_1} u(x_1)\right]^2 + \left[\frac{\partial Y}{\partial X_2} u(x_2)\right]^2 + \ldots + \left[\frac{\partial Y}{\partial X_n} u(x_n)\right]^2}$$

Uncertainty component

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Finding u_c

Important:

The equations on the previous slides are applicable only in the case of non-correlating input quantities!

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