IMAGE EQUALIZATION BASED ON SINGULAR VALUE DECOMPOSITION

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Abstract

In this paper, a novel image equalization technique which is based on singular value decomposition (SVD) is proposed. The singular value matrix represents the intensity information of the given image and any change on the singular values change the intensity of the input image. The proposed technique converts the image into the SVD domain and after normalizing the singular value matrix it reconstructs the image in the spatial domain by using the updated singular value matrix. The technique is called the singular value equalization (SVE) and compared with the standard grayscale histogram equalization (GHE) method. The visual and quantitative results suggest that the proposed SVE method clearly outperforms the GHE method.

Introduction

Contrast is the difference in visual properties that makes an object (or its representation in an image) distinguishable from other objects and the background. In visual perception of the real world, contrast is determined by the difference in the color and brightness of the object with other objects in the same field of view. The human visual system is more sensitive to contrast than absolute luminance; hence, we can perceive the world similarly regardless of the considerable changes in illumination conditions.

If an image is overall very dark or a very bright, the information may be lost in those areas which are excessively and uniformly dark or bright. The problem is how the contrast of an image can be optimized to represent all the information in the input image. Techniques such as histogram equalization, gamma correction, and linear contrast correction have been used to reduce these effects [1] - [4].

In many image-processing applications, the standard grayscale histogram equalization (GHE) method is one of the simplest and most effective primitives for contrast enhancement [5], which attempts to produce an output histogram that is uniform [6]. Not only GHE but also dynamic histogram equalization (DHE) [7] obtained from dynamic histogram specification (DHS) [8] which generates the specified histogram dynamically from the input image, is used for image equalization.

One of the disadvantages of the GHE is that the information laid on the histogram of the image will be totally lost. Demirel and Anbarjafari showed that the histogram of face images can be used for face recognition [9]. In their work it is necessary that if any equalization is used, the pattern of the histogram should not be changed. Fig. 1 (a) shows a low density face image from CALTECH frontal face database [10] and its respective histogram in (b). As it is shown in fig. 1 (d) after the equalization using GHE the shape of the histogram has been totally changed.

In this work, we have proposed a new method of image equalization based on singular value decomposition. The proposed method, which is called the singular value equalization (SVE), preserves the general shape of the histogram and will reduce significantly the loss of information contained in the histogram. Furthermore, the

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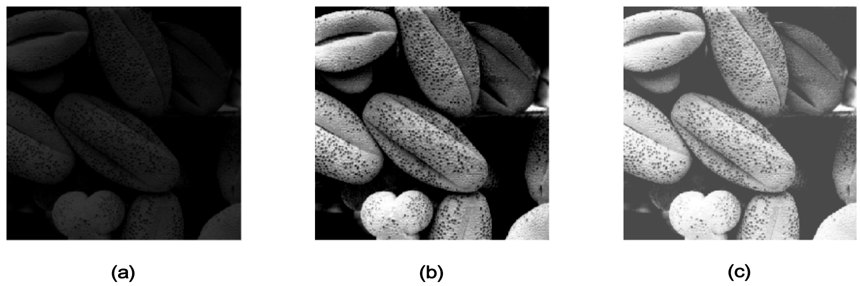

Fig. 2: A low contrast image (a) and its corresponding equalized ones, by using SVE (b) and GHE (c).

proposed method has been compared with GHE method and the visual and quantitative results suggest that the proposed method is superior to the GHE method. The quantitative results are based on the Kullback-Leibler Distance between histograms of the input and processed images.

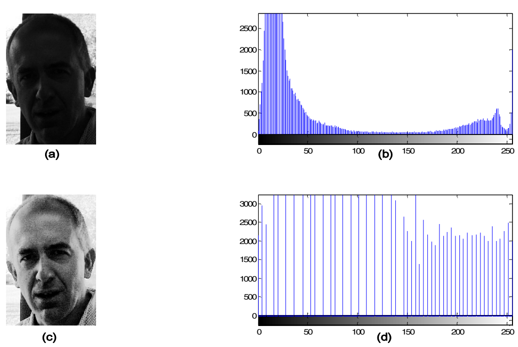

Fig. 1: A face image from the CALTECH face database (a), its histogram (b), the equalized face image using GHE (c) and its respective histogram (d).

Singular Value Equalization

The proposed singular value equalization (SVE) technique is based on the singular value decomposition (SVD). Each image can be represented by a matrix which contains the pixel intensity values. In general, for any image matrix A, the SVD can be defined as:

$$A = U_A \Sigma_A V_A^T \qquad (1)$$

Where U and V are orthogonal square matrices and Σ_A matrix contains the sorted singular values on its main diagonal. Σ_A contains the intensity information of the given image [11], which means that the maximum singular value of Σ_A contributes more than the other singular values. In this paper primary attention is based on the maximum singular value. SVD representation of images has been used in many image processing applications such as image content retrieval [12] and image watermarking [13]. In this paper we have proposed to use singular value representation of an image to equalize the low contrast images. This task can be achieved by normalizing the values in Σ_A matrix as follows. The average intensity value of a gray level image, μ, can be calculated by,

$$\mu = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} a_{ij}}{N \times M} \qquad \left(A = \left[a_{ij} \right]_{M\times N} \right) \qquad (2)$$

where a_{ij} is the intensity value of pixel of the i^{th} row and j^{th} column of image, $\boldsymbol{A}$ with M rows and N columns. It is clear that a low contrast dark image will have $\mu < 128$ and a low contrast bright image will have $\mu > 128$. The objective is to equalize a dark or bright low contrast image in such a way that the mean increases or decreases towards the neighborhood of 8-bit mean gray value 128 respectively. The maximum singular values of a dark image, $\max(\Sigma_D)$, a single gray level image with mean 128, $\max(\Sigma_{128})$, and a bright image, $\max(\Sigma_B)$, can be related to each other as follows.

$$\max(\Sigma_D) < \max(\Sigma_{128}) < \max(\Sigma_B) \qquad (3)$$

Thus we introduce a new image, $\Gamma_{M\times N}$, with a single intensity value that is equal to 128. It is obvious that $\mu_\Gamma=128$. The SVD of this new image is calculated and the maximum singular value ($max(\Sigma_\Gamma)$) is used to calculate the transformation factor, ξ, as follows:

$$\xi = \frac{\max(\Sigma_\Gamma)}{\max(\Sigma_A)} \tag{4}$$

It is straight forward that ξ is greater than 1 if the image A is low contrast with low intensity and is lower than 1 if the image A is low contrast with high intensity. Now a new singular value matrix, $\overline{\Sigma}_A$, can be defined as:

$$\overline{\Sigma}_A = \xi \Sigma_A \tag{5}$$

$\overline{\Sigma}_A$ can be referred as singular value matrix of the equalized image. Using this equalized matrix, a new image, $\overline{A}$, can be constructed by using these three matrices of U_A, $\overline{\Sigma}_A$ and V_A by applying the following equation:

$$\overline{A} = U_A \overline{\Sigma}_A V_A \tag{6}$$

The intensity of the image has been equalized by equalizing the singular value matrix. Thus the proposed method is named singular value equalization (SVE). In the next section visual and quantitative results obtained from SVE based equalized images are given for further discussion.

Experimental Results and Discussions

Fig. 2 illustrates samples of equalized low contrast images using proposed SVE and standard GHE methods. The visual results clearly indicate that the proposed SVE is outperforming the GHE. As it is shown in the fig. 2 the attained image by using SVE is brighter, sharper and the details in the image is more visible than the one attained by using GHE. In order to compare the performance of SVE and GHE, the intensity of an image has been reduced and then both GHE and SVE have been applied to the low contrast image.

Discussion based on visual results mainly depends on human visual perception and that is why we are proposing to use a quantitative metric based on Kullback- Leibler Distance (KLD) between the histograms of the original image and the equalized image.

(a) (b) (c)

Fig. 3: An original image with ill contrast of the storm, Ataturk and one of the authors (a), the result of equalization by using SVE (b) and GHE (c).

KLD is defined to reflect the information divergence between two probability distributions. Table 1 shows the results of comparison between SVE and GHE for different images. As the histogram of the images is not continuous, and also when one connects peaks of the histogram, the resultant curve will not be smooth, hence smoothing is required. A common smoothing technique is the moving average smoothing [14].

As its name suggests, the moving average smoothing operates by averaging a number of points from the curve to generate each point in the new curve. This procedure is repeated so that a moving window of M points is used to calculate the average of the data set. The algorithm takes the mathematical expression [15]:

$$y_i = \begin{cases} \dfrac{\sum_{j=0}^{M-1} x_{i+j}}{M} & 0 < i < N - (M-1) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where x and y are the input and output curves respectively, M is the number of points in the average which is 5 in this work and N is the total number of data points.

After smoothing, the curves can be used to measure the information divergence/loss due to the equalization process by using the KLD as shown in equation (8).

$$\kappa(q,p) = \sum_i q_i \log\left(\frac{q_i}{p_i}\right) \qquad i = 0,1,2,\dots,\beta\text{-}1 \tag{8}$$

where q and p are representing the equalized and original histograms respectively and β is the number of bins for histograms, which is 256 in this work. κ represents the loss of the information.

Table 1: Comparison between SVE and GHE for low contrast images

Image	$\dfrac{\mathrm{KLD}(SVE)}{\mathrm{KLD}(GHE)}$
Pollen image taken from [2]	0.4952
Cameraman image taken from [2]	0.5915
Lena image taken from [2]	0.2021
One of the Authors' images	0.3926
Average of 40 images from CALTECH database	0.8186

The ratio of KLD values in Table 1 shows how much information has been preserved by using SVE while performing the equalization operation. Fig. 3 shows some images with different intensities and their equalized ones. The high performance of SVE is visible in the sense of clearer and sharper output images. Fig. 4 also shows the histograms of the original image and the low contrast images compared with the histograms of the attained images from SVE and the histograms of the attained images from GHE.

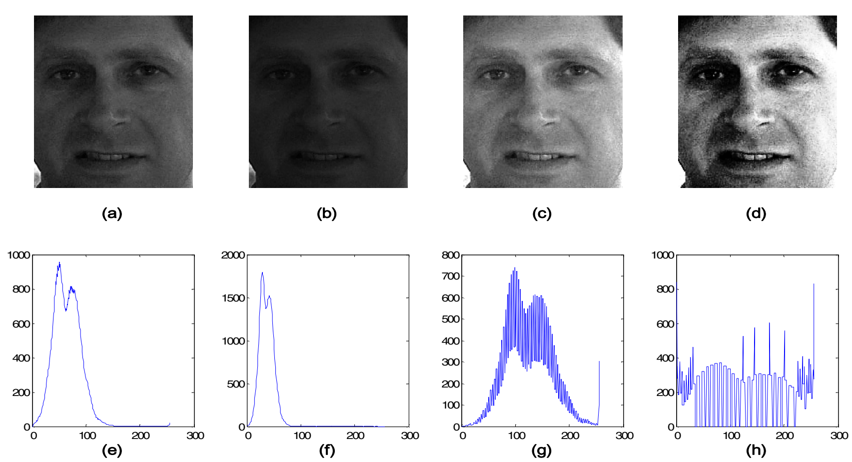

Fig. 4: A face image from Caltech database (a), introduced low density of the same image (b) and the resultant image of SVE (c) and GHE (d) and their respective smoothed histograms (e)-(h).

The histograms in fig. 4 (e)-(h) have been smoothened with using moving average. As fig. 4 (g) shows the equalized image by using SVE has preserved its main statistics such as the number of peaks or local minimums as well as the general pattern of the histogram of the original image shown in fig. 4 (a).

Conclusion

In this paper a novel equalization technique based on SVD has been introduced and implemented. The introduced equalization technique, SVE, has been compared with the standard GHE technique for visual and quantitative performance evaluation. Various tests on low contrast images, taken from different sources, show the superiority of the SVE technique. A quantitative metric based on KLD which shows the information divergence, between two probability distributions, has been employed to reflect the information loss attained by the SVE and GHE techniques after equalizing various low contrast images. The quantitative results supports the visual results that the quality and the information content of the equalized images are better preserved through the proposed SVE technique.

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