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The Teacher's Role in Classroom Discourse: A Review of Recent Research Into Mathematics Classrooms

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Current curriculum initiatives in mathematics call for the development of classroom communities that take communication about mathematics as a central focus. In these proposals, mathematical discourse involving explanation, argumentation, and defense of mathematical ideas becomes a defining feature of a quality classroom experience. In this article, the authors provide a comprehensive and critical review of what it is that mathematics teachers actually do to deal with classroom discourse. Synthesizing the literature around a number of key themes, the authors critically assess the kinds of human infrastructure that promote mathematical discourse in the classroom and that allow students to achieve desirable outcomes. From the findings, they conclude with implications for teachers.

Keywords: discourse processes, mathematics education, qualitative research.

Mathematics, it is widely understood, plays a key role in shaping how individuals deal with the various spheres of private, social, and civil life. Current thinking among researchers and reformers bears this understanding out by putting the spotlight squarely on the social and cultural aspects of mathematical development. The belief is that students' active engagement with mathematical ideas will lead to the development of specific student competencies and identities. It is these competencies and identities that are presumed to make a positive difference in students' life chances and their future civic participation (Ball, 2003). Recent mathematics initiatives have legitimated this kind of thinking by calling for changed classroom communities in which learning rules for manipulating symbols gives way to learning to communicate about and through mathematics. New initiatives such as *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) replace traditional classrooms with "learners talking to each other [and] by groups of students voicing their opinions in whole class discussions" (Sfard, Forman, & Kieran, 2001, p. 1). Talking about mathematics becomes acceptable, indeed essential, in the classroom, and mathematical discussion, explanation, and defense of ideas become defining features of a quality mathematical experience.

New curricula demand a lot from teachers. We were interested in what research has found about the sorts of pedagogies that, through classroom discourse, contribute to students' active engagement with mathematics. Arguably, documenting evidence about pedagogy that produces desirable outcomes for students through classroom discourse is a primary educational necessity (Lampert & Blunk, 1998; Richardson, 2001; Shulman & Shulman, 2004). Responding to the challenge, a number of educational researchers have offered a set of generic pedagogical approaches that appear to hold good across subject domains (e.g., Alton-Lee, 2003; Brophy, 1999; Darling-Hammond & Bransford, 2005). For example, from a seminal publication on generic features of quality teaching (Brophy, 1999), we know that practices and conditions that engage students in thoughtful and sustained discourse can make a difference to learning when the discourse is centered on powerful ideas. We know that teachers who invite students to "develop explanations, make predictions, debate alternative approaches to problems . . . [and] clarify or justify their assertions" (Brophy, 2001, p. 13) have done so through their belief in the rights of all students to have access to education in a broad sense—an understanding of the big ideas of curriculum and an "appreciation of its value, and the capability and disposition to apply it in their lives outside of school" (Brophy, 1999, p. 4).

Identifying effective discursive practices in domain-specific areas, such as mathematics, is a more recent research endeavor (Ball, Lubienski, & Mewborn, 2001; Shulman, 1986; Stein, 2001). As a result, an understanding of what quality mathematics pedagogy looks like, specifically in relation to the vision of communal production and validation of mathematical ideas, is still in its formative stages. To assist, broad guidelines have been provided by the National Council of Teachers of Mathematics (2000) about some of the things that teachers might do to enhance effective classroom discourse: "Effective teaching involves observing students [and] listening carefully to their ideas and explanations" (p. 19). We take these directives as our starting point to report on the features of effective pedagogy that are specific to classroom discourse in mathematics classrooms.

Our review looks critically at recent research on quality mathematics pedagogy that produces desirable student outcomes. The review is timely, given that for many students mathematics is a series of hurdles and challenges—a task met with continued failure and seeming irrelevance. Today, just as in past decades, many students do not succeed in mathematics, are disaffected by it, and continually confront obstacles to engage with the subject. Recent analyses of international test data (e.g., the Program for International Student Assessment, or PISA, of the Organization for Economic Co-Operation and Development, 2004) confirm a trend of systematic mathematics underachievement among particular groups of students. These data point to the dilemma that teachers, schools, and policy makers face when confronted with the realization that schools will cater to increasingly diverse groups of learners. These changing demographics require a wider understanding of what discourse might look like in the classroom in order for it to be effective for all students. The difficulty is compounded in the knowledge that ensuring that mathematical pedagogy is both effective and equitable is not a straightforward matter of importing more general pedagogical cures (see Shulman, 1986).

If characterizing effective and equitable pedagogy is not simply a matter of applying generic approaches, the challenge for those with an interest in mathematics education is to understand what effective pedagogy might look like for all students enrolled in mathematics courses. In this article, we confront that challenge and begin by reiterating what teachers themselves have long known: that enhancing the competencies and identities of all learners, to a large extent, rests with how teachers operationalize the core dimensions of pedagogy (Muijs & Reynolds, 2001; Rowe, 2004). We provide a comprehensive and critical review of what it is that mathematics teachers actually do to deal with classroom discourse in a way that enhances desirable student outcomes. In the next section, we outline the method we used to access our data. We then synthesize the literature, organizing the discussion around a number of themes through which we critically assess the kinds of human infrastructure, noted in the literature, that allow students to achieve mathematical and social outcomes. We conclude with the implications of our review of this large matrix of practice for classroom teachers.

Method of Locating and Assembling Data

Our review looks at research that addresses the following question: What are the characteristics of pedagogical approaches to classroom discourse that produce desirable outcomes for diverse students? The review draws on data from the *Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration* (Anthony & Walshaw, 2007). The synthesis is part of the Iterative Best Evidence Synthesis Programme, established by the Ministry of Education in New Zealand, to deepen understanding from the research literature of what works in education for diverse learners. Our task in that project was to determine what the literature says about quality mathematics teaching for diverse students across sectors. The undertaking involved the engagement of library personnel and advisory and audit groups. It also involved the education community, in general, and the mathematics education community, in particular, which assisted in locating publications in academic journals, theses and projects, and other scholarly work, across sectors, with a focus on mathematics in schools or centers worldwide. The networks we engaged ensured that the synthesis would be inclusive of views from across the community.

In this review, our objective is to provide directions for teachers and other educators about communication in mathematics classrooms. That objective had the effect of excluding discourse studies undertaken in the preschool and tertiary context, with the local community, and in the home. In addition, the literature search was confined to studies undertaken in English-speaking countries. The intent was to access a number of measures that derived from the what, why, how, and under what conditions questions concerning pedagogical approaches that facilitate positive outcomes for all school students. We searched both print and electronic indices, endeavoring to make our search as broad as possible within the limits of manageability. Crossing classroom boundaries, the search took into account relevant publications in the general education literature and in specialist educational areas.

Specifically, the search covered scholarly work in

- key mathematics education literature including all major mathematics education journals (e.g., *Journal for Research in Mathematics Education, Educational Studies in Mathematics, Journal of Mathematics Teacher Education, For the Learning of Mathematics, Journal of Mathematical Behaviour*), international conference proceedings (e.g., International Group for the Psychology of

Mathematics Education, International Congress on Mathematical Education, Mathematics Education Research Group of Australasia publications), and international handbooks of mathematics education (e.g., English, 2002);
- relevant Australasian-based studies, reports, and thesis databases;
- education journals (e.g., *American Educational Research Journal, British Educational Research Journal, Cognition and Instruction, Elementary School Journal, Learning and Instruction*);
- specialist journals and projects, especially those located in the wider education field (e.g., *Journal of Learning Disabilities*); and
- landmark international studies, including the Third International Mathematics and Society Study, PISA, and the U.K. Leverhulme projects.

This search strategy led us to a large body of literature that served as a starting point for identifying effective pedagogical practices that link with student outcomes.

Selecting the Evidence

In our first pass through the literature, we noted that many studies offered detailed explanations of student outcomes yet failed to draw conclusive evidence about how those outcomes related to specific teaching practices. Others provided detailed explanations of pedagogical practice yet made unsubstantiated claims about, or provided only inferential evidence for, how those practices connected with student outcomes. These particular studies did not satisfy our selection criteria precisely, because we were searching for studies that offered not simply descriptions of pedagogy and descriptions of outcomes but rigorous explanations for close associations between the two.

To be more specific, in our search we were looking at pedagogy that made links with student outcomes, both academic and social. In particular, we took the National Research Council's (2001) understanding of mathematical proficiency as

- conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- strategic competence: the ability to formulate, represent, and solve mathematical problems;
- adaptive reasoning: ability for logical thought, reflection, explanation, and justification; and
- productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

We added to these academic outcomes a range of other outcomes that relate to affect, behavior, communication, and participation. The outcomes include

- a sense of cultural identity and citizenship;
- a sense of belonging;
- contribution;

- well-being;
- exploration; and
- commonly held values, such as respect for others, tolerance, fairness, caring, diligence, nonracist behavior, and generosity.

Apart from our focus on outcomes, in our selection of data, we were also taking care to eliminate studies that identified teacher effectiveness solely through either (a) teacher uptake of curriculum reforms or (b) the use of test results (Koehler & Grouws, 1992). Just as we would want to believe that reformers' visions are being realized, we have long known that teachers do not always implement them in ways that were intended by curriculum designers (Millet, Brown, & Askew, 2004). Similarly, we found that best practice descriptions and explanations, closely tied to high-stakes assessment, failed to tell the whole pedagogical story. Indeed, as our review clarifies, teaching is a complex activity—some teaching approaches and classroom arrangements produce differential results from one setting to another.

Searching for studies that made connections between pedagogy and desirable outcomes opened us up to a body of literature comprising different communities, different factors, and different locations. Our search strategy demonstrated a "willingness to consider all forms of research evidence regardless of methodological paradigms and ideological rectitude, and [a] concern in finding contextually effective appropriate and locally powerful examples of 'what works' . . . with particular populations, in particular settings, to particular educational ends" (Luke & Hogan, 2006, p. 175). In dealing with this diversity, we endeavored to capture differences in the context, practices, and ways of thinking of researchers and to report on the research in a way that makes the original evidence as transparent as possible. We included many different kinds of evidence that take into account human volition, program variability, cultural diversity, and multiple perspectives. Each study, characterized by its own way of looking at the world, has led to different kinds of truth claims and different ways of investigating the truth.

We included studies that

- focused on classroom discourse in mathematics classrooms in schools;
- provided a description of the context, the sample, and the data;
- offered details about the particular pedagogy and the specific student outcomes;
- connected research to relevant literature and theory;
- used methods that allowed investigation of the pedagogy–outcome link; and
- yielded findings that illuminated what did or did not work for particular students.

We looked at the ways different pieces of data meshed together and determined the plausibility, coherence, and trustworthiness of the interpretation offered. Assessments about the quality of research depended on the nature of the knowledge claims made and the degree of explanatory coherence between those claims and the evidence provided. In short, for a study's inclusion in this review, we were assessing the explanatory power of the stated pedagogy–outcome link. When assessing the nature and strength of the causal relations between pedagogical approaches and learning outcomes, we were guided by Maxwell's (2004) categorizations of two types of explanations of causality. The first type, the *regularity view of causation,* is based on observed regularities across a number of cases. The second type, *process-oriented explanations,* perceives "causality as fundamentally referring to the actual causal mechanisms and processes that are involved in particular events and situations" (p. 4). Attention to both types of explanation of causality provided the impetus to include research reports of empirical studies, from very small single site settings to large-scale longitudinal experimental studies.

Framing and Organizing the Data

We conceptualized classroom discourse as nested within an evolving systems network. In this system, the teacher and the students are mutually constituted through the course of interactions. The notion of a close relationship between social processes and conceptual development draws its inspiration from the work of post-Vygotskian activity theorists such as Davydov and Radzikhovskii (1985). It is a proposal that is fundamental to Lave and Wenger's (1991) well-known social practice theory, in which the notions of "a community of practice" and "the connectedness of knowing" are central features and in which individual and collective knowledge emerge and evolve within the dynamics of the spaces people share and within which they participate. Learning mathematics involves activity within a community. It entails "how to generate . . . ideas, how to express them using words and symbols, and how to justify to oneself and to others that those ideas are true" (Carpenter, Franke, & Levi, 2003, p. 1).

Engeström (e.g., 1999) has provided a conceptual grounding for explaining community in activity systems. His framework, focusing on a collective activity system as the primary unit of analysis set within a larger network of systems, offers a way of analyzing roles, responsibilities, and resources in relation to an activity. The theory is able to deal with cultural/historical communal systems, such as classrooms settings, that are driven by common motives, while simultaneously being in constant movement and confronted with internal contradiction. Figure 1 shows Engeström's model of the human activity system.

Applying the model to our discussion on the role of pedagogy in classroom discourse, the object of attention is focused on the learner, the subject is the teacher, and the outcomes are those discussed earlier that are deemed desirable for the student. The rules are those that are either implicit or articulated, or both, within the community of the classroom. The division of labor is that made visible between teacher and student. Tools and signs include the teacher's knowledge (both pedagogical content knowledge and knowledge of students) and the resources (both human and material) that are available in the classroom.

One of the important things that derives from the model is the contingency of student outcomes on a network of interrelated factors and environments: Outcomes are not so much *caused* by pedagogy as they are *occasioned* by those practices. In effect, social and academic outcomes are occasioned by a complex web of relationships around which knowledge production and exchange revolve. This means that the processes operating at the macrolevel of the system, involving policy and institutional governance, create a context for the work that teachers do within the

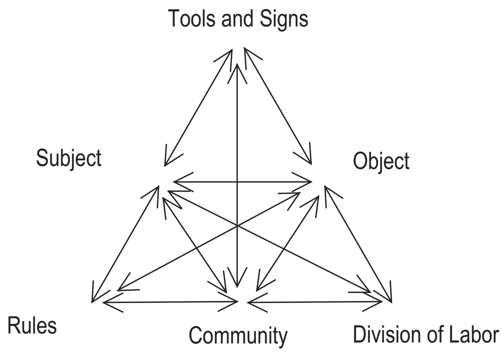

FIGURE 1. *Engeström's cultural/historical activity theory.*

microlevel of the classroom. Given the scope of the system, the features of quality practice necessarily constitute a large matrix of practice, consisting of multiple dimensions and complex relationships between its parts. Studies have revealed how interactions between elements within the system profoundly influence the students' constructions of mathematical ways of knowing (e.g., Boaler, 2003; Watson, De Geest, & Prestage, 2003).

Quality teaching, then, is a joint enterprise, founded on material, systems, human, and emotional support, as well as on the collaborative efforts of teachers to make a difference for all learners (Coburn, 2005). In making a difference through classroom discourse, teachers shift students' cognitive attention toward making sense of their mathematical experiences, rather than limiting their focus to procedural rules. According to Yackel and Cobb (1996), students become less engaged in solutions to problems than in the reasoning and thinking that lead to those solutions. It is through the patterns of interaction and discourse created in the classroom that students develop a mathematical disposition—ascribing meaningfulness to one another's attempts to make sense of the world. Learning about other ways to think about ideas, to reflect, and to clarify and modify thinking is fundamental to moving learning forward. Carpenter et al. (2003) maintain that the very nature of mathematics presupposes that students cannot learn mathematics with understanding without engaging in discussion and argumentation.

But more talk in classrooms does not necessarily enhance student understanding. Better understanding is dependent on particular pedagogical approaches, purposefully focused on developing a discourse culture that elicits clarification and produces consensus within the classroom community. In reviewing the work undertaken in this area, we found that a number of activities related to pedagogical practice came to the fore. These include (a) participating rights and obligations, (b) articulating thinking, (c) fine-tuning mathematical thinking through language, and (d) shaping mathematical argumentation. We use this activity cluster to organize the literature on classroom discourse in mathematics. Each activity serves as a point of discussion in the Results section. Together, the activities provide insight into definitions of effective domain-specific pedagogy relating to classroom discourse in mathematics classrooms.

Results

Participating Rights and Obligations

There is now a large body of empirical and theoretical evidence that demonstrates the beneficial effects of participating in mathematical dialogue in the classroom (e.g., Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Fraivillig, Murphy, & Fuson, 1999; Goos, 2004; Hicks, 1998; Hiebert et al., 1997; Kazemi & Franke, 2004; Lampert & Blunk, 1998; Manouchehri & Enderson, 1999; McClain & Cobb, 2001; Mercer, 2000; O'Connor, 1998; Sfard & Kieran, 2001; Simon & Blume, 1996; White, 2003; Whitenack, Knipping, & Kim, 2001; Wood, Williams, & McNeal, 2006). What these researchers have demonstrated is that effective instructional practices demand students' mathematical talk.

Yackel and Cobb (1996) made the important observation from their research that the daily practices and rituals of the classroom play an important part in how students perceive and learn mathematics. These practices and rituals include the rights and obligations of mathematical participation. Cobb, Wood, and Yackel (1993) reported that students create "insider" knowledge of mathematical behavior and discourse from the norms associated with those daily practices. This knowledge evolves as students take part in the "socially developed and patterned ways" (Scribner & Cole, 1981, p. 236) of the classroom. By scaffolding the development of those patterned ways, the teacher regulates the mathematical opportunities available in the classroom.

However, it is a major challenge for many teachers to include classroom discourse as an integral part of an overall strategy of teaching and learning (Hicks, 1998; Lampert & Blunk, 1998). How and when does the teacher set up practices that will enable students to participate in mathematics discussions? Wood (2002) researched six classes over a 2-year period, investigating the patterns of interaction in the classrooms. From data collected on a daily basis during the first 4 weeks of school, Wood examined the ways in which the six teachers set up the social norms for classroom interaction. Further data were gathered at a later date to compare and contrast discursive interactions when the same instructional activity took place in different classrooms. Wood found variation in students' ways of seeing and reasoning, and these were attributed in the first place to the particular differences established in classrooms early in the year concerning *when* and *how to contribute* to mathematical discussions and *what to do as a listener.* The discourse principles propping up classroom participation regulated the selection, organization, sequencing, pacing, and criteria of communication. Consistent with findings reported by a number of other researchers (e.g., Dekker & Elshout-Mohr, 2004; Ding, Li, Piccolo, & Kulm, 2007; Gillies & Boyle, 2006; Webb, Nemer, & Ing, 2006), Wood's analysis showed that varying classroom expectations and obligations served to create marked differences in the cognitive levels demanded of the students. Furthermore, participation obligations put boundaries around the opportunities for students to share their ideas and to engage in mathematical practices (Ding et al., 2007; Fuchs et al., 1997; Veenman, Denessen, van den Akker, & van der Rijt, 2005; Webb et al., 2006).

In an investigation into elementary (primary) school students' responses to tasks requiring explanations of commutativity, Anthony and Walshaw (2002) noted that many students did not know how to explain their mathematical ideas; indeed, several students were decidedly ill at ease with the proposition that they share their thinking with others. Bicknell (1998) studied students' written explanations and justifications for mathematical assessment tasks. The study showed that many of the 36 Year 11 (age 15–16) research participants were uncertain about what was expected when asked to explain their ideas.

Woodward and Irwin (2005) described how a particular teacher involved in their study made a significant contribution to students' mathematical development. She did this by listening attentively to her students' queries and explanations and by asking them to justify their answers. The researchers recorded one of many occasions during which she structured students' mathematical practice: "Before you write it down I want you to justify it to your partner. So if you say there's eight queens your partner needs to say, 'How do you know that there's eight queens?'" (p. 803). By using such pedagogical strategies, Woodward and Irwin made the point that the teacher was able to develop mathematical ways of doing and being in all her students.

Pedagogical practices that create opportunities for students to explain their thinking and to engage fully in dialogue have been reported in research undertaken by Steinberg, Empson, and Carpenter (2004). In a study from their Cognitively Guided Instruction Project, classroom discussion was central to a sustained change in students' conceptual understanding. Over a period of a few months, the teacher integrated particular pedagogical strategies focused on probing and interpreting student understanding and on generating new knowledge. During the study, she developed a working consensus with all members of the classroom community about the form of, and social roles within, her changed instructional processes.

Honoring students' contributions is an inclusive pedagogical strategy. Yackel and Cobb (1996) found that classroom teachers who facilitate student participation and elicit student contributions and who invite students to listen to one another, respect one another and themselves, accept different viewpoints, and engage in an exchange of thinking and perspectives exemplify the hallmarks of sound pedagogical practice. Nathan and Knuth (2003) studied a teacher's Year 1–2 classroom over a 2-year period. In particular, under investigation were the classroom expectations and obligations concerning who might speak, when, and in what form and what listeners might do. Progressively, the teacher learned to facilitate students' participation in classroom interactions and to ensure that students shared their thinking and listened attentively to each other. For example, the teacher told the class, "If you don't have an opinion, will you try and get one so we can keep this [discussion] going a little longer" (p. 195). As a result of the changed norms of participation, there was a marked increase in student contribution.

Framing this discussion in terms of a collective activity system, Figure 2 represents the teacher's first action in a practice focused on whole-class participation and active engagement in dialogue. It represents a beginning step in a wider system of pedagogical activity within the classroom to ensure desirable outcomes for all students in a classroom. The activity is a central element of the teacher's goal-directed action, aligning well with recent curricula reforms that set great stock on classroom discourse for enhancing learning. The particular action requires teachers to clarify, establish, and enforce discourse participation rules within the classroom community. In this action, the divisive nature of the teacher's and students'

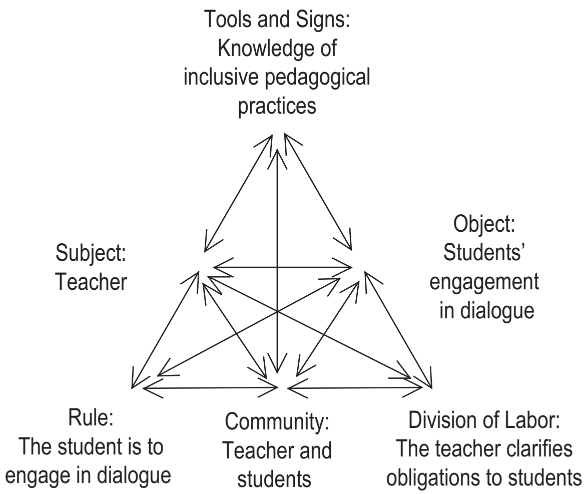

FIGURE 2. *Activity 1: Clarifying discourse participation rights and responsibilities.*

roles is imminently visible. The accomplishment of the outcome of this first action—to engage all students in dialogue—is dependent on a shared understanding of the importance of dialogue and the sharing of mathematical ideas.

A number of situations might arise in which the outcome is not fully realized. For example, a number of studies have reported that some students, more than others, appear to thrive in whole-class discussions. In their respective research, Baxter, Woodward, and Olson (2001) and Ball (1993) found that highly articulate students tend to dominate classroom discussions. Typically, low academic achievers remain passive; when they do participate visibly, their contributions are comparatively weaker and their ideas sometimes muddled. Teachers sometimes unwittingly create inconsistent social norms of participation. Planas and Gorgorió (2004) investigated social interactions at the beginning of the high (secondary) school year in a mathematics classroom in Spain with a high percentage of immigrant students. Unlike local students, immigrant students were not permitted to participate in mathematical argumentation and hence did not have personal experience of how participation could help clarify and modify thinking. The researchers observed the teacher's subtle, systematic refusal of immigrants' attempts to explain and justify their strategies for solving problems. As Planas and Gorgorió reported, the reduced social obligations and lesser cognitive demands placed on these students had the effect of excluding them from full engagement in mathematics and hence constrained their development of a mathematical disposition. Teaching for inclusion ensures that participation in classroom discussion is safe for all students.

A number of other researchers (e.g., Boaler, Wiliam, & Brown, 2000; Clark, 1997) have also reported students' differential access to productive engagement in classroom discourse and, importantly, to the mathematical identities that result. From their longitudinal Effective Teachers of Numeracy study involving six U.K. schools, Boaler and colleagues documented that lower streamed classes followed a protracted curriculum and experienced less varied teaching strategies. This curriculum polarization had a marked effect on the students' sense of their own mathematical identity. The findings of these researchers are supported by other international research into the detrimental effects of such grouping arrangements on the teaching and learning of students in lower streams (e.g., Gamoran, 1992; Zevenbergen, 2005).

Lubienski (2002), as teacher–researcher, focused on the inclusive aspects of classroom dialogue when she compared the learning experiences of students of diverse socioeconomic status (SES) in a seventh-grade classroom. She reported that higher SES students believed that the patterns of interaction and discourse established in the classroom helped them learn other ways of thinking about ideas. The discussions helped them reflect, clarify, and modify their own thinking and to construct convincing arguments. However, in Lubienski's study, the lower SES students were reluctant to contribute, stating that the wide range of ideas contributed to the discussions confused their efforts to produce correct answers. Their difficulty in distinguishing between mathematically appropriate solutions and nonsensical solutions influenced their decisions to give up trying. Pedagogy, in Lubienski's analysis, tended to privilege the ways of being and doing of high-SES students. In a similar way, Jones's (1991) study showed that the discursive skills and systems knowledge that are characteristic of high-SES families align them favorably with the pedagogy that is operationalized within school settings. Set in the New Zealand context, Jones provided conclusive evidence that Pasifika girls were unwittingly penalized by the sorts of instructional and noninclusive approaches taken by classroom teachers.

Differentiating Between Responses and Supporting Students' Thinking

In the previous section, we noted that a large number of researchers have reported the cognitive advantages to students through their participation in mathematical discussion. By expressing their ideas, students are able to make their mathematical reasoning visible and open for reflection. Not only does the expression of student ideas provide a resource for teachers, informing them about what students already know and what they need to learn; the ideas also become a resource for students themselves—challenging, stimulating, and extending their own thinking. However, many of the same researchers who elevate student articulation of mathematics thinking caution, simultaneously, that teachers who sustain discursive interactions with a view merely to keeping the conversation going do not necessarily move students' thinking forward.

For example, in their exploration into the teaching practice of one teacher, McClain and Cobb (2001) reported that although the teacher in their study honored students' differing ideas, strategy sharing within the classroom community was used as an end to itself. The practice did not attempt to differentiate between the mathematical integrity of students' ideas. These findings support the earlier work reported by Doyle (e.g., Doyle & Carter, 1984) on classroom participation. Specifically, Doyle found that teachers use the strategy of "accepting all answers" as a way of simply achieving student cooperation in an activity. Mercer (1995) demonstrated that a pedagogical practice that does not attempt to synthesize students' individual contributions tends to constrain the development of mathematical thinking.

A pedagogical approach that is able to move students' thinking forward involves significantly more than developing a respectful, trusting, and nonthreatening climate for discussion and problem solving. It involves socializing students into a larger mathematical world that honors standards of reasoning and rules of practice (Popkewitz, 1988). O'Connor and Michaels (1996) put it this way:

> The teacher must give each child an opportunity to work through the problem under discussion while simultaneously encouraging each of them to listen to and attend to the solution paths of others, building on each others' thinking. Yet she must also actively take a role in making certain that the class gets to the necessary goal: perhaps a particular solution or a certain formulation that will lead to the next step. . . . Finally, she must find a way to tie together the different approaches to a solution, taking everyone with her. At another level—just as important—she must get them to see themselves and each other as legitimate contributors to the problem at hand. (p. 65)

Effective pedagogy is inclusive and demands careful attention to students' articulation of ideas. Franke and Kazemi (2001) made the important claim that an effective teacher tries to delve into the minds of students by noticing and listening carefully to what students have to say. Yackel, Cobb, and Wood (1998) provided evidence to substantiate the claim. They reported on the ways in which one Year 2 teacher listened to, reflected on, and learned from her students' mathematical reasoning while they were involved in a discussion on relationships between numbers. Analyses of the discussion revealed that her mathematical subject knowledge, and her focus on listening, observing, and questioning for understanding and clarification, greatly enhanced her understanding of students' thinking.

Jaworski (1994), similarly, offered firsthand accounts of listening and responsive action in the classroom. Contrary to scholarly critique that claims such practice is impossible on the grounds that the metacognitive activity involved assumes more time than the classroom could possibly offer, Jaworski (2004) provided evidence of teachers noticing and then acting knowledgeably as they interacted at critical moments in the classroom when students created a moment of choice or opportunity.

Other researchers have also provided evidence of the critical role of the teacher in listening to students and orchestrating mathematical discourse. In a study undertaken within a heterogeneously grouped seventh-grade mathematics classroom, Manouchehri and Enderson (1999) found an overwhelming occurrence of student talk and interaction. A more in-depth analysis of the discursive interactions revealed that the teacher provided responsive rather than directive support, all the while monitoring student engagement and understanding. She did this through careful questioning and purposeful interventions and with a view toward shifting the students' reliance from her toward the support and the challenge of peers with varying levels of skills and understanding. The teacher's strategy was not aimed at structuring learning by organizing students' behavior. Rather, as Manouchehri and Enderson clarified, her primary objectives were to

- facilitate the establishment of situations in which students had to share ideas and elaborate on their thinking (e.g., Would anyone else like to add anything to S13's explanation? Could you show that to us on the board? That is an excellent question. Does anyone want to have a shot at it?);

- help students expand the boundary of their exploration (e.g., What do you think class? Do you think that this formula would work all the time for all the rows? Why don't you extend the sequence and see if there is a pattern?);
- encourage students to make connections among different discoveries and develop a deeper understanding of the interrelationships among the patterns that students identified (e.g., I wonder if we can find out how these 2 patterns are related?); and
- invite multiple representations of ideas (e.g., Is there another way of representing this?). (Manouchehri & Enderson, 1999, p. 219)

A number of other studies have investigated precisely what it is that teachers do to gain access to students' thinking while engaged in dialogue with their students. White (2003) explored how two teachers used classroom discourse to teach third graders mathematics, looking at how the discourse enhanced the educational experiences of the diverse student populations in the two research classrooms and how it influenced the mathematical thinking of the students. The teachers were part of a larger project, Increasing the Mathematical Power of All Children and Teachers. Classroom vignettes illustrated one of four themes that emerged from the classroom discourse: (a) valuing students' ideas, (b) exploring students' answers, (c) incorporating students' background knowledge, and (d) encouraging student-to-student communication. The teachers engaged all students in discourse by first monitoring their participation in discussions and then deciding when and how to encourage each to participate. By actively listening to students' ideas and suggestions, they demonstrated the value they placed on each student's contribution to the thinking of the class. The teachers encouraged their students to give critical feedback on each other's responses and asked them to reveal their assessments of each other's ideas by giving a thumbs up or thumbs down signal. In one of the classrooms for limited-English-speaking students, this proved to be a particularly useful strategy for getting members to share their views with the class.

Fraivillig et al. (1999) reported on the discursive exchange of ideas that took place within a Year 1–2 classroom. What was particularly effective was the way the teacher purposefully sustained the discussions. In particular, she knew when to "step in and out" (Lampert & Blunk, 1998) of the classroom interactions. Knowing when to intervene is a rich resource for teachers focused on making a difference in students' learning (Chamberlain, 2005; Cobb et al., 1993; Gipps, McCallum, & Hargreaves, 2000; J. Hill & Hawk, 2000; Khisty & Chval, 2002; Sadler, 1989; Wiliam, 1999).

The literature reveals that a sensibility for redirecting the discussion to ensure that important mathematical ideas are being developed is dependent on a range of pedagogical content knowledge skills. Turner and colleagues (1998, 2002) found that what distinguished high-involvement Year 5 and Year 6 classrooms was the engagement of the teachers in forms of instruction that allowed them to negotiate meaning through "telling" tailored to students' current understandings. They shared and then transferred responsibility so that students could attain greater autonomy. They also tended to foster motivation by sparking curiosity and by supporting students' goals. In these classrooms, telling was followed by a pedagogical action that had the express intent of finding out students' understandings and interpretations of the given information.

In another study, directed at investigating the way in which a teacher interacted with her fifth-grade Latino students (and with English-language learners), Khisty and Chval (2002) found that the teacher's focus on mathematical talk and meaning enabled the students to develop mathematical reasoning in significant ways. She facilitated learning through questioning that was concerned less about teacher exposition and more about the perceptions held by her students. The teacher opened up the discussion with each interaction, and by making use of the responses articulated, she was able to lend structure to their mathematical meanings.

However, in other studies, teachers demonstrated uncertainty about precisely how to lend structure to students' mathematical thinking (Ding et al., 2007; Dekker & Elshout-Mohr, 2004; Fuchs et al., 2007; Veenman et al., 2005; Webb et al., 2006). Knight (2003) interviewed six teachers and observed the verbal support that teachers provided in their elementary school classes. The study revealed that out of the 349 examples of verbal responses made by teachers recorded in the lessons, most (83%) took the form of an expression of encouragement or praise. Knight reported that the teachers were often unaware of the high frequency of such responses and their automated nature. Only 17% of the feedback reflected on the cognitive development of the students.

Classroom research at the high school level (e.g., Ruthven, 2002; Watson, 2002) has also shown that much of the teacher support that students receive does not assist students "to study mathematics and think mathematically" (Ruthven, 2002, p. 189). For example, Watson (2002) reported that teaching mathematics to low-attaining students in high school "often involves simplification of the mathematics until it becomes a sequence of small smooth steps which can be easily traversed" (p. 462). Frequently, teachers took students through the chain of reasoning, and students merely filled in the gaps with the arithmetical answer or low-level recall of facts. This "path smoothing," it was found, did not lead to sustained learning precisely because the strategy deliberately reduced a problem to what the learner could already do—with minimal opportunity for cognitive processing. This kind of pedagogical support reinforces the view that if students sit and do nothing for long enough, the teacher will change the level of intellectual engagement so that the task can be completed with minimal effort (Anthony, 1996).

Ding et al. (2007) emphasized that teachers who provide verbal support with path-smoothing assistance, steering students toward a particular solution (product-help assistance), are less effective in improving students' thinking than are teachers whose interventions scaffold the students toward possible lines of approaches to solving the problem (process-help assistance). These researchers suggested that support should build on and order the mathematical thinking resources provided by the student group to allow important mathematical ideas to surface.

Expanding on this aspect, Cobb et al. (1993) reported that effective teachers in their research influenced the course of the dialogue by framing students' interpretations and solutions as topics for discussion. Valuing and shaping students' mathematical contributions served these important functions:

- It allowed students to see mathematics as created by communities of people.
- It supported students' learning by involving them in the creation and validation of ideas.

- It helped students to become aware of more conceptually advanced forms of mathematical activity.

Researchers (e.g., Hiebert et al., 1997; O'Connor & Michaels, 1996) have found that relevant and meaningful teacher responses to student talk involves drawing out the specific mathematical ideas set within students' methods. Probing into student understanding provides teachers with the opportunity to model engagement within a mathematical, multivoiced community, with a view toward advancing students' understanding of appropriate mathematical conventions. Reframing student talk in mathematically acceptable language provides teachers with the opportunity to enhance connections between language and conceptual understanding. In reporting on their work, O'Connor and Michaels used the term "revoicing" to mean the repeating, rephrasing, or expansion of student talk in order to clarify or highlight content, extend reasoning, include new ideas, or move discussion in another direction.

Forman and Ansell (2001) found that in classrooms where revoicing is used, "There is a greater tendency for students to provide the explanations . . . and for the teacher to repeat, expand, recast, or translate student explanations for the speaker and the rest of the class" (p. 119). The teacher in their study used repetition to highlight the particular claims and ideas of individual students, developed the understandings implicit within those ideas, negotiated meaning to establish the veridicality of a claim, and used students' original ideas as a springboard for developing related new knowledge in whole-class discussions. In another study, Cobb et al. (1993) described how one teacher's feedback was able to shape student participation, giving it the cultural nuances of mathematical talk:

> [The teacher] reformulated their explanations and justifications in terms that were more compatible with the mathematical practices of society at large and yet were accepted by the children as descriptions of what they had actually done. Thus rather than funnelling the children's contributions, the teacher took the lead from their contributions and encouraged them to build on each others' explanations as she guided conversations about mathematics. As a consequence, the mathematical meanings and practices institutionalized in the classroom were not immutably decided in advance by the teacher but, instead, emerged during the course of conversations characterized by . . . a genuine commitment to communicate. (p. 93)

In her research, Sherin (2002) found that in classroom exchanges of ideas, typically, teachers negotiate between three areas of knowledge: their understanding of subject matter, their perception of curriculum materials, and their personal theories of student learning. As they weave between these three areas of knowledge and as they deepen their own understanding of them, effective teachers are able to increase students' levels of mathematical knowledge. The negotiation that takes place as teachers reflect-in-action draws on a rich history of personally established ways of thinking and being and applying knowledge flexibly. In particular, such teachers are able to adapt and modify their routine practices and, in the process, contribute to the development of new pedagogical routines and new knowledge about subject matter.

Adapting and modifying routine practice requires professional reflecting-in-action. H. Hill, Rowan, and Ball (2005) found from observations in their study of instructional improvement that reflecting-in-action involves a moment-by-moment synthesis of actions, thinking, theories, and principles. Askew and Millet (in press) observed, in their Leverhulme Numeracy Research Program, that teachers who were able to develop student mathematical understanding were those who had a sound base of subject knowledge. This knowledge informed their on-the-spot decision making during classroom interactions. It informed decisions about the particular content that the students would learn, the activities students carried out, and how students engaged with the content.

Of course, reflecting-in-action can take place either in the classroom (Sherin, 2002) or beyond (Mewborn, 1999; Wood, 2001). It involves, in the first instance, listening carefully to students' expressions of mathematical content (Davis, 1997). Effective teachers, in listening, notice significant mathematical moments and respond appropriately (Schifter, 2001; Sherin, 2001). A study undertaken by Davies and Walker (2005) reported on the use of classroom videotapes to focus teachers' attention on observing what was said and done in the classroom. Through viewing video clips of their own practices, teachers reported an enhanced awareness, in the sense of noticing significant mathematical moments. The new awareness meant that they were more likely to respond to the mathematical learning needs of their students.

Taking all these subactivities together in our second action of classroom discourse, the object of attention has moved from student talk, per se, to the content of students' talk. The activity takes place within a complex network of teacher practice focused on classroom discourse (shown in Figure 3). The outcome is a clear articulation of students' ideas and is reached by purposeful differentiation between students' ideas and the scaffolding of students' thinking. Such scaffolding takes place through careful listening and attentive noticing on the part of the teacher, through responsive rather than directive support, through a sensibility to the cultural capital and background knowledge brought to the classroom by diverse students, through revoicing and questions from the teacher and other students, and through teacher sensitivity about when to step in and when to step out of classroom discussions. An important mediating factor in this action is the continuing creation of a division of labor between student and student. Other students and the teacher have a visible presence within the community, focused on the articulation of thinking with a view toward enhancing new knowledge.

Without responsive pedagogical support, the desired outcome of students' advancement of mathematical ideas becomes elusive. The corollary is also true. Too much support is counterproductive to learning. This point is illustrated by Woodward and Irwin (2005) in their comparative study of two teachers. For one of the teachers, the researchers recorded numerous instances of too much feedback or "teacher lust" (Maddern & Court, 1989). Although this teacher had created a positive learning environment and shown a keen desire for talk to occur in the classroom, actual mathematical talk was minimal. Cognitive space was limited by the lack of pause times for thinking, and students were occasionally "talked over." Specifically, students did not have the opportunity to learn and speak the language of mathematicians. By not providing students with opportunities to engage in mathematical discourse, this teacher prevented students from expressing what they were learning. Cognitive space was also constrained by the teacher in a study by Khisty and Chval (2002). Although the teacher in the study had done many of the

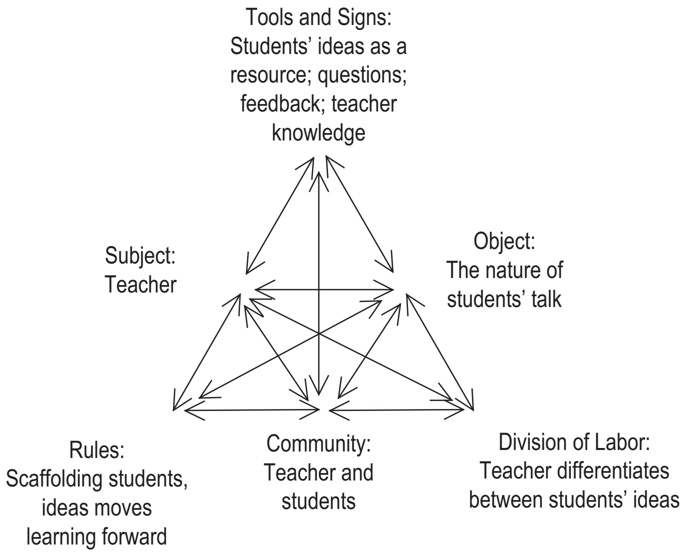

FIGURE 3. *Activity 2: Scaffolding students' ideas to move thinking forward.*

"proper" things by, for example, contextualizing mathematics in a story that had relevance to the students, by developing a role-playing activity to assist with conceptual understanding, and by forming small working groups, the teacher nevertheless failed to provide students with the disciplinary meanings associated with mathematical knowledge.

Fine-Tuning Mathematical Thinking Through Language

So far, we have considered a number of aspects of pedagogy that enable students to engage in meaningful dialogue about mathematics. Yet engagement in effective classroom discourse is "a complex process that combines doing, talking, thinking, feeling, and belonging" (Wenger, 1998, p. 56). As we have seen, engagement in discourse that successfully advances students' understanding demands a respectful exchange of ideas, teacher listening, attentiveness, and reflection-in-action. It also involves familiarizing students with mathematical convention. Effective teachers are able to bridge students' intuitive understandings with the mathematical understandings sanctioned by the world at large. In this section, we focus on language as our third activity. We do so in the belief that language plays a central role in building bridges between disciplinary and everyday ideas. Language constructs meaning for students as they move toward modes of thinking and reasoning characterized by precision, brevity, and logical coherence (Marton & Tsui, 2004). In particular, the teacher who makes a difference for diverse learners is focused on shaping the development of novice mathematicians who speak the precise and generalizable language of mathematics.

McChesney (2005) explored students' contributions in low- and middle-band classes at the junior high school level. McChesney noted that teachers who established classroom communities in which there was access to discursive resources were able to significantly support students' mathematical activity. Her research clearly demonstrated a direct relationship between the quality of teacher–student interaction and students' negotiation of mathematical meaning. Over time, students' contributions, which were initially marked by informal understandings, began to appropriate the language and the understandings of the wider mathematical community.

It is through the take-up of conventional language that mathematical ideas are seeded. Khisty and Chval (2002) argued that effective teachers are able to set up an environment in which conventional mathematical language migrates from the teacher to the students. Hence, the meanings that students construct ultimately descend from those captured through the kind of language the teacher uses. To enculturate students into the mathematics community, effective teachers share with their students the conventions and meanings associated with mathematical discourse, representation, and forms of argument (Cobb & Yackel, 1996; Wood, 2002). In short, the responsibility for distinguishing between terms and phrases and sensitizing students to their particular nuances weighs heavily with the teacher, who profoundly influences the mathematical meanings made by the students.

Competency in mathematics demonstrates control over the specialized discourse (Gee & Clinton, 2000). But the specialized language of mathematics can be problematic for learners. Particular words, grammar, and vocabulary used in school mathematics can hinder access to the meaning sought and the objective understood for a given lesson. Words, phrases, and terms can take on completely different meanings from those assumed in everyday contexts. Walkerdine (1988), for example, reported the difficulties young students encounter in establishing the mathematical meaning of *less than* and *more,* given their idiosyncratic meanings within specific home practices. Similarly, Christensen (2004) provided evidence that mathematical words can sometimes assume quite different meanings in everyday life. He documented the difficulties encountered by Maori language students in using *mua* and *muri* for number sequencing and in using *whakamua* and *whakamuri* for forward and backward counting.

Developing this point for Australian students, Sullivan, Zevenbergen, and Mousley (2003) found that students with a familiarity of standard English (usually students from middle-class homes) had greater access to school mathematics. As the teachers in their study said, the students were able to "crack the code" of the language being spoken. One teacher of students from non-English-speaking backgrounds made the point about meanings of words:

> You need to reinforce: "Tell me what I mean when I say estimating?" or "Where are some things that you estimate?" Ground it in their world because for a child for whom English is not their first language, if there are numbers they'll be right, but if you say "estimating" they won't have a clue what that might mean. (p. 118)

Mathematical language presents difficulties for students, in general, and presents certain tensions in multilingual classrooms, in particular. In our reading of the literature, we found a number of studies that investigated the specific challenges of teaching mathematics in multilingual contexts (e.g., Adler, 2001; Khisty, 1995; Moschkovich, 1999, 2002). Neville-Barton and Barton (2005) looked at these tensions as experienced by Chinese Mandarin-speaking students in New Zealand schools. Their investigation focused on the difficulties that could be attributable to limited proficiency with the English language. It also sought to identify language features that might create difficulties for students. Two tests were administered 7 weeks apart. In each, one half of the students sat the English version and the other half sat the Mandarin version, ensuring that each student experienced both versions. There was a noticeable difference in their performances on the two versions. On average, the students were disadvantaged in the English test by 15%. What created problems for them was the syntax of mathematical discourse, in particular, prepositions, word order, and interpretation of difficulties arising out of the contexts. Vocabulary did not appear to disadvantage the students to the same extent. Importantly, Neville-Barton and Barton found that the teachers of the students in their study had not been aware of some of the student misunderstandings.

Like the students in the study undertaken by Neville-Barton and Barton (2005), students from Samoa and Tonga, in Latu's (2005) research, had difficulty with syntax. Word problems involving mathematical implication and logical structures such as conditionals and negation were a particular issue for students in senior mathematics classes. They also found technical vocabulary, rather than general vocabulary, to be problematic. Latu noted that English words are sometimes phonetically translated into Pasifika languages to express mathematical ideas when no suitable vocabulary is available in the home language. The same point was made by Fasi (1999) in his study with Tongan students. Concepts such as absolute value, standard deviation, and simultaneous equations, and comparative terms such as *very likely, probable,* and *almost certain,* have no equivalent in Tongan culture, whereas some English words, such as square (*sikuea*), have multiple Tongan equivalents. The suggestion is that special courses in English mathematical discourse be delivered with the express intent of connecting the underlying meaning of a concept in English with the students' home language.

Fasi (1999) investigated the discursive approaches of two teachers, one Samoan and the other Tongan, both of whom had been educated in their native countries before moving to New Zealand to complete their higher education. He found that the teachers switched between the language of instruction and the learners' main language in order to explain and clarify the concepts to students. Similarly, Adler (1998), Clarkson (1992), and Dawe (1983), as well as Setati and Adler (2001), found evidence of language switching (code switching) by teachers for bilingual students, particularly when students could not understand the mathematical concept or when the task level increased. Code switching involved words and phrases as well as sentences and tended to enhance student understanding.

In this third activity within a constellation of discursive classroom practices, and as depicted in Figure 4, the object of attention is students' mathematical language. Based on familiarization with students' intuitive and culturally specific understandings of words and expressions, and on the teacher's own knowledge of mathematical language, the teacher, as subject, shapes an understanding within the classroom community of the language of mathematics, sensitizing students to particular meanings of words and terms and to the syntax of mathematical discourse. Fundamental to the outcome of this activity is the rule that mathematical language takes precedence over cultural or intuitive understandings. In this activity, primary importance is placed on the teacher's knowledge of her or his own students and knowledge of mathematics in mediating students' understanding of mathematical

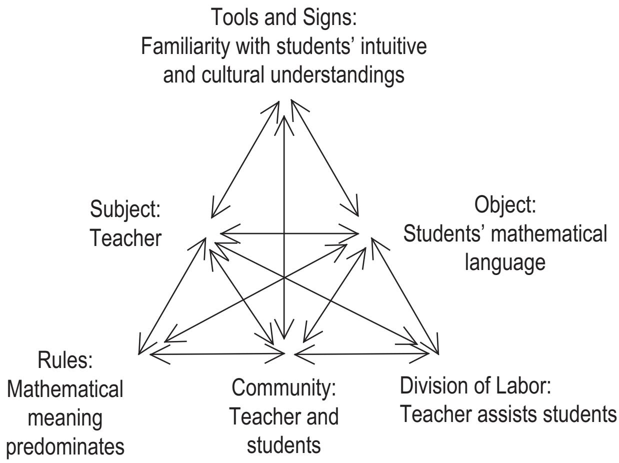

FIGURE 4. *Activity 3: Fine-tuning mathematical thinking through language.*

language. Thus, the division of labor between teacher and student is as apparent as it was in Activities 1 and 2.

Shaping Mathematical Argumentation

We have now looked at the approaches teachers take to fine-tune thinking through language. But mathematical language involves more than technical vocabulary. It also encompasses the way it is used within mathematical argumentation. The positive effects of providing regular opportunities for students to engage in argumentation have been well documented (e.g., Carpenter & Lehrer, 1999; Cobb, Boufi, McClain, & Whitenack, 1997; Empson, 2003; Goos, 2004; Kazemi & Stipek, 2001; Lampert, 1990; McClain & Cobb, 2001; O'Connor, 2001; White, 2003; Wood & McNeal, 2003; Zack & Graves, 2002). These researchers have provided evidence that students should have the opportunity and space to, for example, interpret, generalize, justify, and prove their ideas, as well as to critique the ideas of others in the class.

Many researchers have found that pedagogical practices that allow students to engage in these activities greatly enhance the development of their mathematical thinking. The development of thinking depends not so much on the frequency of exchange structures but on the extent to which students are regarded as active epistemic agents. Developing students' thinking also enhances the view that students hold of themselves as mathematics learners and doers. In particular, O'Connor and Michaels (1996) highlighted the importance of shaping mathematical argumentation by fostering students' involvement in taking and defending a particular position against the claims of other students. They pointed out that this instructional process depends on the skillful orchestration of classroom discussion by the teacher. The skill "provides a site for aligning students with each other and with the content of the academic work while simultaneously socializing them into particular ways of speaking and thinking" (p. 65).

Stigler (1988) looked at teachers who had developed the skill of shaping students' mathematical responses. He compared the pedagogical approaches of Japanese and American teachers and found that Japanese teachers spend more time than American teachers aligning students' mathematical ideas with disciplinary conventions to which they are accountable. However, scaffolding argumentation practices is a highly complex activity (Anghileri, 2006; Taylor & Cox, 1997). It is complex because teachers and students are "negotiating more than conceptual differences. . . . They are building an understanding of what it means to think and speak mathematically" (Meyer & Turner, 2002, p. 19).

For students, discussion, debate, and critique are all learned strategies. Sfard and Kieran (2001) emphasized that "the art of communicating has to be taught" (p. 70). In pedagogical terms, it involves more than sustaining discursive interactions with a view to keeping the conversation going; rather, it requires nudging the conversation in mathematically enriching ways. Pedagogical approaches that have been shown to be effective include the modeling of high-level performance by the teacher and/or capable peers; the making of conceptual connections (Kazemi & Franke, 2004); the provision of appropriate time for exploring ideas and making connections (Stein, Grover, & Henningsen, 1996); the encouragement of student self-monitoring (Pape, Bell, & Yetkin, 2003); and a sustained press for explanation, meaning, and understanding (Fraivillig et al., 1999).

Fraivillig et al. (1999) developed a conceptual framework for describing the ways by which teachers fine-tune students' mathematical thinking through the cognitive structure they provide. The three key pedagogical components identified in their Advancing Children's Thinking framework are eliciting, supporting, and extending. Eliciting involves promoting and managing classroom interactions, supporting involves assisting individuals' thinking, and extending captures those practices that work to advance students' knowledge. Fraivillig et al. found that when the teachers in their study elicited, supported, and extended students' thinking, they tended to move classroom conversations in mathematically enriching ways. Specifically, they clarified mathematical conventions, and they arbitrated between competing conjectures. In short, they picked up on the critical moments in discursive interactions and took learning forward.

A pedagogical approach that extends students' thinking in a classroom environment that perpetuates classroom agreement about the importance of classroom discussion and debate gets to the heart of student thinking. Yackel and Cobb (1996) reported from their research that to do this the teacher must first construct the norms for what constitutes a mathematically acceptable, different, sophisticated, efficient, or elegant explanation. These are the norms that were found to regulate the content and direction of mathematical argumentation and govern the learning opportunities and ownership of knowledge made available within the classroom.

Zack and Graves (2002) reported that teachers who develop student argumentation and enhance learning are themselves active searchers and enquirers into mathematics. O'Connor's (2001) classroom research highlighted how one teacher, through purposeful listening, facilitated a group of students toward a mathematical solution. The students took varying positions toward the solution and attempted to support those positions with evidence. The teacher made her contribution by challenging the students' claims by using counterexamples. In doing so, she scaffolded thinking by providing a predictable structure for inquiry through which she enacted an expectation regarding ownership, self-monitoring, and argumentation.

Goos (2004) described how a high school mathematics teacher developed his students' mathematical thinking through scaffolding the processes of inquiry. For his part, the teacher orchestrated mathematical events by first securing student attention and participation in the classroom discussion. Specifically, the "teacher call[ed] on students to clarify, elaborate, critique, and justify their assertions. The teacher structured students' thinking by leading them through strategic steps or linking ideas to previously or concurrently developed knowledge" (p. 269). In a series of lesson episodes, Goos provided evidence of how the teacher pulled learners "forward into mature participation in communities of mathematical practice" (p. 283). As the year progressed, the teacher gradually withdrew his support to push students toward more independent engagement with mathematical ideas. For their part, the students responded by completing tasks with decreasing teacher assistance and by proposing and evaluating alternative solutions.

In another study, Stein et al. (1996) identified the factors associated with the maintenance of high-level cognitive activity. In 64% of the tasks in the study that remained high level, a sustained press for justifications, explanations, and meaning, as evidenced by teacher questions, comments, and feedback, was a major contributing factor. This factor was frequently accompanied by the teacher's or capable students' modeling of competent performance—often in the format of a class presentation of a solution. Presentations modeled the use of multiple representations, meaningful exploration, and appropriate mathematical justification. In many cases, a press for understanding resulted in successive presentations illustrating multiple ways of approaching a problem.

The press for understanding is an aspect of quality mathematics pedagogical practice highlighted by many researchers (e.g., Kazemi & Franke, 2004; Morrone, Harkness, D'Ambrosio, & Caulfield, 2004; Stein et al., 1996). When a teacher "presses a student to elaborate on an idea, attempts to encourage students to make their reasoning explicit, or follows up on a student's answer or question with encouragement to think more deeply" (Morrone et al., 2004, p. 29), the teacher is getting a grip on what students actually know and is providing an incentive for them to enrich that knowledge. Morrone and colleagues provide us with examples of effective teacher talk: (a) "So in this situation how did you come up with $^{18}/_{27}$ and $^{18}/_{30}$?" and (b) "When can you add the way we're adding, using the traditional algorithm, finding the common denominator? When does that make sense? Several of you started, the first thing you did was add, and you ended up, what did you end up with, with $^{19}/_{16}$. What does that mean? When can you do that? When does it make sense to add that way?" (p. 33).

We now consider how this fourth action of teachers' work is depicted as an activity in the classroom. As in Activities 1, 2, and 3, Activity 4 is both nested within a discursive complex and takes as its object the student. All mathematics pedagogy is goal directed, irrespective of an individual teacher's viewpoint of her or his work. Indeed, the activity of pedagogy would not exist without students. Day-to-day teaching practice in the classroom is focused on assisting students to learn and to become numerate with a future focus on responsible citizenship. Hence, it is not surprising to note that in Activity 4 (depicted in Figure 5) the

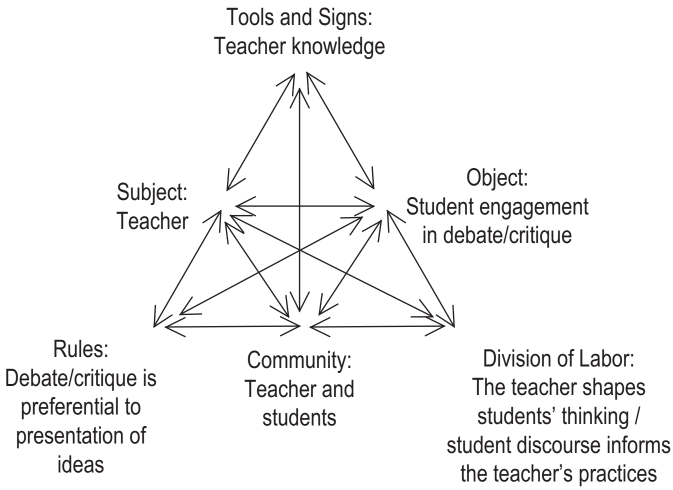

FIGURE 5. *Activity 4: Shaping mathematical argumentation.*

teacher again assumes the dominant subject position. In doing so, the teacher provides a sense of continuity and coherence to the actions involved in discursive interactions focused on student learning. The teacher-as-subject position works to sustain the division of labor established earlier between teacher and students. The difference between the division of labor depicted in this model with divisions of labor embodied in traditional models of classroom activity can be explained in this way: Traditional models assume a close link between high levels of teacher control and minimal student discourse, whereas Activity 4, like the previous activities we have discussed, assumes that a high level of pedagogical effectiveness presupposes productive discursive exchanges in the classroom community.

There are a number of aspects of teachers' practice that serve as useful mediators to student involvement focused on the outcome of student argumentation. Fundamental to students' successful participation in debate and defense of ideas is the cognitive structure that the teacher provides. Subordinate units of analysis include the teacher's construction of norms for and modeling of acceptable and, indeed, elegant explanations and justifications; skillful orchestration of the discussion; sustained press for meaning; and resolution of competing students' claims and the mounting of a challenge to students' claims. All these aspects of practice are instrumental to the desired outcome.

Implications for Teachers

What can teachers learn from the literature? What does this review tell us about how mathematics teachers create the conditions for productive classroom discourse? Although there is no definitive formula that explains how teachers do this, the sort of conditions that contribute to enhancing student outcomes have been noted in the literature. We have structured these aspects according to four interrelated and interactive activities. Active student engagement in discourse is influenced by a complex array of factors, many of which are determined, in the first instance, by the pedagogical affordances and constraints within the participation rights and responsibilities established in the classroom. Specifically, the opportunity for learning is influenced by what students are helped to coproduce through dialogue. The effective use of classroom discourse makes students' mathematical reasoning visible and open for reflection. In an environment where ideas are shared, students' own ideas become resources for their own learning. But more than that: Their explanations stimulate, challenge, and extend other students' thinking.

Teachers who are effective have been shown to establish classroom spaces that are truly conducive to sharing. They work at developing interrelationships that create cognitive and physical spaces for students to develop their mathematical and cultural identities. In classroom arrangements, creating such spaces depends a great deal on creating a hospitable environment that makes it possible to reason, communicate, reflect on, and critique ideas. It also depends on creating opportunities for students to do this through classroom discussion. Teachers who work toward an outcomes-based agenda emphasize purposeful and thoughtful discourse and provide opportunities for sharing this in the classroom.

At the same time, research quite clearly demonstrates that pedagogy that is focused solely on the acceptance of all answers and solutions does not strike at the core of what mathematics discourse truly entails. Support for significant mathematical thinking is dependent on teachers who not only hear but listen attentively to the mathematics in students' talk. A context that supports the growth of students' mathematical identities and competencies builds on students' responses, shapes the reasoning and thinking to an appropriate level, and moves ideas and solutions toward a satisfactory conclusion.

This review draws our attention to the fact that teachers who have the intention of developing student understanding will not necessarily produce the desired effect. Unless teachers make good sense of the mathematical ideas they hear in class, they will not develop the flexibility they need for spotting the golden opportunities and wise points of entry that they can use for moving students toward more sophisticated and mathematically grounded understandings. Reflecting on the spot and dealing with contested mathematical thinking demand sound teacher knowledge. Importantly, the way in which teachers manage multiple viewpoints is very much dependent on what they know and believe about mathematics and on what they understand about the teaching and learning of mathematics. A successful teacher of mathematics will have both the *intention* and the *effect* to assist pupils in making sense of mathematical topics. Moreover, the effective teacher is able to make sense of students' conceptual understandings and is able to determine where those understandings might be heading.

Teachers should take heart that in providing opportunities for students to explore mathematics through a range of discursive contexts they contribute to the enhancement of social and cognitive engagement. The most effective settings provide a balance between opportunities for students to benefit from teacher telling and students' involvement in discussion and debate. The activities that teachers plan, and the sorts of mathematical discussions that take place around those activities, are crucially important to learning. Effective teachers plan their classroom discussions with many factors in mind, including the individual student's knowledge and experiences and the participation norms established in the classroom.

Extensive research in this area has found that effective teachers develop their planning to allow students to develop habits of mind whereby they can engage with mathematics productively and make use of appropriate language to support their understanding.

Quality teaching at all levels ensures that mathematical discussion is not simply a time filler but is focused instead on the solution of a genuine mathematical problem. The most productive discourse is that which allows students to access important mathematical concepts and relationships, to investigate mathematical structure, and to use techniques and notations appropriately. Research provides sound evidence that when teachers employ classroom discourse for these purposes over sustained periods of time, they provide students with opportunities for success, they present an appropriate level of challenge, they increase students' sense of control, and they enhance students' mathematical disposition.

Students' informal mathematical knowledge typically arises out of their everyday activities and cultural backgrounds, and quality discursive interactions build on this knowledge. Studies have provided conclusive evidence that teaching that is effective is able to bridge students' intuitive understandings and the mathematical understandings sanctioned by the world at large. Arguably, language plays a central role in that process. The teacher who makes a difference for diverse learners is focused on shaping the development of novice mathematicians who speak the precise and generalizable language of mathematics. By focusing on language in this way, the teacher profoundly influences the mathematical meanings made by the students in the class.

A classroom context that supports students' growing awareness of themselves as legitimate participants in the production of mathematical knowledge creates a space for all heritages. Although many researchers have shown that classroom exchanges of mathematical ideas can provide the context for social and cognitive engagement, others have cautioned that limited-English-speaking students are less inclined to share their thinking in group processes. Teachers need to be aware that other students, too, do not appear to thrive in class discussions. A personal reluctance to participate and the low social obligations and cognitive demands unwittingly placed by some teachers on low-achieving students have the effect of excluding them from full engagement in mathematics.

Consistently emphasized in research is the fact that classroom work is made more enriching when discussion involves the respectful exchange of ideas, when teachers ensure that this exchange is inclusive of all students, and when the ideas put forward are (or become) commensurate with mathematical conventions and curricular goals. It involves a sustained press for justification and explanation, as well as a sensitivity to know when to intervene and when to step out. Revoicing students' mathematical proposals can contribute to a respectful exchange of ideas within the classroom community. In the most effective exchange of ideas, the teacher is able to orchestrate discussion and argumentation and facilitate dialogue, not only for the development of mathematical competencies and identities but also to ensure important and desirable social outcomes. In short, the classroom community participates in a "microcosm of mathematical practice" (Schoenfeld, 1992), learning how to appropriate mathematical ideas, language, and methods and how to become apprentice mathematicians.

Gaps and Omissions

We note that recent research in mathematics education, in the main, centers on interrelationships within teaching practice. In particular, today's research tends to emphasize the coconstruction of knowledge. Hence, it explores the social context of teaching and learning. The research we have reviewed explores the social context of teaching through both quantitative and qualitative research designs. We make these observations with the caveat that we do not claim to have covered the field completely and absolutely. Some omissions have arisen because it has not been possible to include every piece of research evidence about classroom discourse. We acknowledge, too, that a number of research projects of international repute have not been included because they focus on other areas of mathematics education. Other gaps have arisen simply because gaps exist in the literature. To date, we do not know as much about quality classroom discourse at the high (secondary) school level as we do about the elementary (primary) level. The literature also lacks longitudinal, large-scale studies that explore the links between classroom discourse and learning. Such research is important for understanding teachers' work and the impact of reform.

Conclusion

Finding out what kinds of contexts and communities support mathematics discourse for outcomes-focused pedagogy is crucial for education. Teachers who implement pedagogical reform, in relation to classroom discourse, must inevitably focus on developing community, ensuring that those within the community are given opportunities to talk about, support, and nurture each other's learning. The review we have provided here represents a systematic and credible evidence base about quality discourse in mathematics classrooms and explains the sort of pedagogical approaches that lead to improved engagement and desirable outcomes for learners from diverse social groups.

The search through the literature focused attention on different contexts, different communities, and multiple ways of thinking and working. As a result of that search, we have presented and interpreted instances of deliberate efforts by teachers to do the best possible job for their students. Our review sketches out in broad strokes evidence of quality mathematics teaching. The evidence draws on the histories, cultures, languages, and practices found in mathematics classroom contexts. In a distinctive approach, it considers a wide range of research evidence drawn from diverse research methodologies, to investigate what works successfully in practice. It is committed to the development of knowledge and pedagogical competencies suited for multicultural societies, cosmopolitan citizenship, and educational change.

In theorizing classrooms as activity systems, our review has found that effective pedagogy is motivated by a deeply communal concern to do the best possible job for students. The subject is the teacher, and the student is the object of pedagogical work within the collective activity system. Hence, the nature of discourse in the classroom is not a dialogue between equals, no matter how equitable the goals of classroom community might be and no matter how skillful the teacher is at exploiting and scaffolding the nature of discourse so that knowledge appears to be coconstructed.

Pedagogical practice is general and individual, bringing both generic motives and unique imperatives. Our focus on the aspect of discourse and scaffolding of student engagement has revealed a range of teacher generic skills, knowledge, and dispositions that contribute to advancing students' knowledge. These are aspects that have been found to make a difference to all students, irrespective of subject area. These pedagogical factors shape how, and with what effect, subject content is taught and learned. However, like Brophy (1999), we wish to emphasize the complexity of teaching. Student outcomes are contingent on a range of cultural scripts and imperatives. Each classroom context brings its own characteristics to the wider network of pedagogical activity systems. Although our review has surveyed the literature on mathematics classroom discourse, it is important to note that classroom discourse will gain positive effect only when there is a strong cohesion between all the various elements and interrelated contingencies of a teacher's work. In other words, the facilitation of productive classroom discourse is part of a larger activity network that allows students to develop habits of mind to engage with mathematics productively and make use of appropriate mathematical tools to support their understanding.

This proviso stems directly from our conceptualization of mathematics teaching as part of a nested activity system. A teacher, for us, is not so much an isolated entity but a unity nested within other unities such as the particular students within the classroom and the school. In this conceptualization, teaching is influenced by adaptive rather than additive factors and by interactive rather than isolated variables. This means that the outcomes of teaching are contingent on a network of interrelated factors, conditions, and environments. These are the factors and conditions that shape how, and with what effect, mathematics is taught and learned. Within the nested activity system, teachers modify and transform their pedagogical practices: They adapt their practices in relation to their personal understandings and to the system-level processes of the school and other educational institutions.

Several important ideas follow from this. First, creating discourse opportunities in the classroom is a complex activity. Quality teaching is not simply the fact of "knowing your subject" or the condition of "being born a teacher." Second, by nesting teaching within a systems network, we cannot claim that teaching causes student outcomes. Understanding this prevents us from romanticizing the teaching of mathematics; it also prevents us from reducing effective practice to prescriptive decree. But if student outcomes are not *caused* by teaching practices, they can at least be *occasioned* by those practices. And in this review, we have offered important insights from research about how that occasioning might take place. Certain patterns have emerged that have enabled us to foreground ways of doing and being that mark out an effective pedagogical practice. Each aspect, of course, constitutes but one piece of evidence and must be read as accounting for one variable, among many, within the teaching nested system. Taking all these aspects together allows us to envisage what quality classroom discourse might look like.

Our review has deepened our understanding of mathematics discursive practices in many ways. We found that inclusive classroom partnerships are fundamental to effective teaching. Facilitating respectful and patterned interactions in the classroom contributes to the enhancement of students' aspirations, attitudes, and achievements. Teachers who set up conditions that are conducive to classroom discussion come to understand their students better. Students benefit, too, and the ideas put forward in the classroom become rich resources for knowledge. Through students' purposeful involvement in discourse, through listening respectfully to other students' ideas, through arguing and defending their own positions, and through receiving and providing a critique of ideas, students enhance their own knowledge and develop their mathematical identities.

The findings highlight that both the cognitive and material decisions that teachers make, in relation to classroom discourse, significantly influence learning. Consistently emphasized in research is the fact that discursive contexts that are rich in cognitive (and social) experiences for all students support the development of creative thinking and problem-solving skills. Teachers who are able to provide such contexts simultaneously increase students' sense of control and develop valuable student mathematical dispositions. Classroom work is made more enriching when discussion involves the coconstruction of mathematical knowledge through the respectful exchange of ideas. When teachers work at developing inclusive partnerships for the exchange of ideas, they ensure that the ideas put forward are, or become, commensurate with mathematical convention and curricular goals.

We would like to think that this evidence-based review of mathematics pedagogy classroom discourse provides a starting point and a language for discussing reform, innovation, and change in mathematics pedagogy. Subject-specific research into classroom discourse is still in its formative stages, but we are slowly developing a sensibility from research, such as that of Wood and colleagues (2006), about how discourse patterns influence mathematical thinking and learning. When searching for studies to review that offered a "detailed look at how [teachers'] actions played out in the classroom and how students were involved in this" (Blanton & Kaput, 2005, p. 435), we have learned a lot about what discourse works, as well as how and why it works. What has been highlighted for us is the enormous complexity of teaching. Policy makers, experienced and new teachers, and other educators often fail to appreciate the full span of pedagogical practice. Our hope is that this review not only makes that complexity explicit but also provides ways of doing things that will assist teachers and policy makers in their endeavors.

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